**GEORGIA INSTITUTE OF TECHNOLOGY**

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# RESEARCH PROJECT INITIATION

Date: [illegible]

Project Title: [illegible]

Project No: [illegible]

Principal Investigator [illegible]

Sponsor: [illegible] Science Foundation

Agreement Period: From 3/1/77 Until [illegible]/77
[illegible]

Type Agreement: Grant No. [illegible]

Amount: [illegible]

Reports Required:

[illegible] Final Report

Sponsor Contact Person (s):

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3 (6-71)

GEORGIA INSTITUTE OF TECHNOLOGY

OFFICE OF CONTRACT ADMINISTRATION

# SPONSORED PROJECT TERMINATION

Date: 1/10/79

Project Title: Research ~~Initiation~~ - Energy Absorption in Lightly Damped Free Standing Towers

Project No: E-20-668

Project Director: Dr. B. J. Goodno

Sponsor: National Science Foundation

Effective Termination Date: 10/31/77

Clearance of Accounting Charges: 10/31/77

Grant/Contract Closeout Actions Remaining:

- _ Final Invoice and Closing Documents
- X Final Fiscal Report (See Imp. Notice No. 68)
- _ Final Report of Inventions
- _ Govt. Property Inventory & Related Certificate
- _ Classified Material Certificate
- _ Other ______________

Assigned to: Civil Engineering (School/Laboratory)

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A-4 (3/76)

# RESEARCH INITATION
# ENERGY ABSORPTION IN LIGHTLY DAMPED FREE-STANDING TOWERS

ANNUAL REPORT

April 1975 - March 1976

NSF Grant ENG 75-10320

Dr. Barry J. Goodno
School of Civil Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

ANNUAL PROGRESS SUMMARY

NSF Grant ENG 75-10320

Work on the subject grant entitled "Research Initation - Energy Absorption in Lightly Damped Free-Standing Towers" was begun in April 1975 and has proceeded along three primary paths: (1) collection and review of references; (2) computer program development; (3) and collection of data on the prototype structure.

The original reference list has been expanded considerably to include a number of recent contributions to the technical literature. Reports of investigators studying the wind profile and wind induced response of large self-supporting towers in Canada and in Europe have provided valuable information on the dynamic properties of large tower structures and on instrumentation, measurement and data reduction techniques. Other papers concerned with studies of small steel transmission towers have also been reviewed for completeness.

Development of the computer program for static analysis of large trussed towers using substructuring techniques is nearing completion. Several problems related to data handling and use of auxiliary storage remain to be solved. A test program, DYNATRUSS, for the linear dynamic analysis of space trusses using modal analysis is complete and has been thoroughly checked. Routines for numerical integration of the equations of motion and assembly of the damping matrix are under development at the present time.

The 1100 feet (335m) Channel 17 television tower in Atlanta, Georgia, is serving as the prototype structure for this study. Assembly of the tower data including joint coordinates, member sizes and material properties was a formidable task but is now essentially complete. This data is being card punched at present in preparation for generation of a structure plot from the joint coordinates in order to check overall structure geometry.

It is anticipated that computer program development will be completed by June 1976. Summer 1976 will be devoted to error checking, parameter studies, and field measurements at the site of the prototype structure.

ANNUAL RESEARCH SUMMARY

NSF Grant ENG 75-10320

Research effort on the subject grant entitled "Research Initiation-Energy Absorption in Lightly Damped Free-Standing Towers" has been directed along the three paths discussed briefly in the Annual Progress Summary report, namely: (1) Literature Survey; (2) Computer Programs; (3) and Prototype Structure Data Collection.

Literature Survey: Continual review of the literature has uncovered several papers of importance and relevance to this study. Of particular importance are the reports of investigators concerned with the wind-induced dynamic response of a number of large self-supporting towers: the Munich Television Tower (290m) in Germany, the Emley Moore Television Tower (330m) in London, England, and the CN Tower (450m) in Toronto, Canada. Although differing structurally from the trussed steel television tower (height=335m) which serves as the prototype in this study, these other large towers exhibit the general type of behavior to be expected in the prototype structure and will serve as an important basis of comparison for results (vibration frequencies and damping) generated in this study. A series of studies of small latticed steel towers in Hawaii (45 to 122m) also provides comparative data for use here. In addition field measurement and data reduction techniques detailed in these earlier studies may be of use in on-site investigations of the dynamic behavior of the prototype structure to be conducted later.

A number of references on the general character of wind loading on structures and an appropriate analytical description of wind induced loading were also reviewed.

Computer Programs: A considerable amount of activity has been (and continues to be) directed toward development of the computer programs for static and dynamic analysis of free-standing trussed tower structures. The flowchart for program TOWER3 which performs a substructure analysis of an arbitrarily-supported space truss structure is nearly complete. Stiffness terms associated with degree-of-freedom indices at any number of arbitrarily-selected joints can be retained in the substructure forward elimination sequence to permit general selection of overall structure degrees-of-freedom. Instability due to the planar configuration of tower faces is also accounted for in the program. Tributary masses of structural members are lumped at the retained degrees-of-freedom to form the structure mass matrix. The tower damping matrix is assumed to be a linear combination of the mass and stiffness matrices (i.e., proportional damping in which the mass and stiffness matrices are scaled to produce levels of damping that are consistent with measured values for tower structures - usually less than 1% of critical) plus a damping matrix containing equivalent viscous damping coefficients for the discrete energy-absorbing, add-on devices.

Once the mass, damping, and stiffness matrices are assembled, the equations of motion can be solved for the time - dependent response of the structure to general piecewise-linear forcing functions (wind, earthquake) in the three orthogonal structure directions. A test program, DYNATRUSS, which computes and plots the three-dimensional response of a space truss structure using the normal mode method, is complete, has been thoroughly checked, and is ready to be incorporated into program TOWER3.

Several small programs were also written to generate and card punch the joint coordinates and member information for the prototype structure.

At the completion of this task, the joint and member information will be used with the general purpose program ICES STRUDL-II to generate several plots of the structure as a check on overall structure geometry. STRUDL will also be used to produce a reduced structure stiffness matrix for the tower that will be used to obtain preliminary estimates of the dynamic properties and response of the tower. This information will be used later to error-check program TOWER3.

Prototype Structure Data Collection: The 1100 feet (335m) Channel 17 television tower in Atlanta, Georgia, which serves as the prototype structure in this study must be completely described to structural analysis programs STRUDL-II and TOWER3 referred to above. Hence, considerable effort was devoted to collecting and cataloging all joint coordinates, member sizes and material properties, and member connectivity data for the prototype structure. Initial attempts to secure this data from tower structural engineering consultants proved unsuccessful but further investigation revealed that the original tower plans and drawings showing later structural modifications were on file at the Georgia Building Inspection Division offices in Atlanta. Thorough study of the tower drawings has produced the requisite data, and card punching and checking of this data are under way at the present time.

## MAJOR RESEARCH ACCOMPLISHMENTS

NSF Grant ENG 75-10320

Engineers are admittedly at a loss when it comes to detailed predictions of damping in slender self-supporting tower structures which are among the most highly stressed Civil Engineering structures. Several investigators, however, have made dynamic measurements at the site of existing towers and have found that damping values as low as 0.5% of critical are not uncommon in these slender frameworks. It has been the aim of the present study to determine the number, distribution, size, and arrangement of discrete energy absorbing devices required to significantly increase the overall level of damping and thereby reduce the wind and earthquake induced response of such lightly damped structures. Fortunately, a large steel trussed tower in Atlanta, Georgia, is available for study and has been designated the prototype structure. On-site determination of the dynamic properties and response of this 1100 feet (335m) tower is planned for the near future. Comparison of experimental and computer-generated results for this structure will serve to calibrate the analytical model under development at the present time. Finally, the model will be used in parameter studies which seek to determine the effect of discrete damping devices on overall tower response.

A significant amount of the total project effort so far has been expended on the development of the computer programs for the static and dynamic analysis of general space truss structures of which the free-standing tower is a special case; and a considerable amount of time and effort has been devoted to obtaining, cataloging, and checking descriptive data on the prototype structure. The accomplishments in these two areas of activity are the subject of this report.

The flow chart for program TOWER3 which performs a static substructure analysis of an arbitrarily-supported space truss structure is nearly complete. Several unique features of this computer program include the ability to handle (1) planar joints, (2) general location of structure supports, and (3) arbitrary specification of overall structure degrees-of-freedom in independent substructures during the forward elimination sequence. A parallel programming effort has produced working subprograms for the linear dynamic analysis of space trusses by mode-superposition procedures. Joint displacement, member force, and reaction-time histories are printed and plotted for independent, piecewise-linear forcing functions, such as wind or earthquake in the structure x, y, and z directions. Selected routines for dynamic analysis calculations will be incorporated into program TOWER3 in the near future.

The other area of major accomplishment concerns the collection of descriptive data such as joint coordinates and member properties for the prototype structure. Drawings showing the original structure and later structural modifications have been thoroughly studied, and joint and member data is being card-punched at the present time for later use as input data for the program TOWER 3.

## PERSONNEL

NSF Grant ENG 75-10320

The personnel associated with the research program and their contributions are summarized below.

B. J. Goodno, Assistant Professor, Principal Investigator.

Project director; development of static and dynamic analysis computer programs; collection of data for prototype structure.

Hernan P. Torres, Graduate Student Assistant.

Development of substructure analysis computer program; cataloging of tower data.

Kenneth Gram, Graduate Student Assistant.

Cataloging of tower data; general computer programming assistance; literature survey assistance.

## TECHNICAL PAPERS, REPORTS, AWARDS

NSF Grant ENG 75-10320

None submitted or received to date.

E-20-666

RESEARCH INITIATION

# ENERGY ABSORPTION IN LIGHTLY DAMPED FREE-STANDING TOWERS

Final Project Report

Submitted to the
National Science Foundation

by


Georgia Institute of Technology
School of Civil Engineering
Atlanta, Georgia 30332

Barry J. Goodno, Principal Investigator



Grant ENG75-10320

April 1975 - December 1978

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**PART I–PROJECT IDENTIFICATION INFORMATION**

| ion and Address | 2. NSF Program | 3. NSF Award Number |
|---|---|---|
| rgia Institute of Technology<br>anta, Georgia 30332 | Structural, Materials and Geotechnical | ENG75-10320 |
| | 4. Award Period<br>From April 75 To Oct 77 | 5. Cumulative Award Amount<br>$16,900. |

Title

earch Initiation - Energy Absorption in Lightly Damped Free-Standing Towers

**PART II–SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)***

Guyed and free-standing latticed steel towers, widely used as radio and television s, are among the most highly-stressed and dynamically sensitive civil engineering tures. One of the principal goals of this research program was to develop an effi- linear model and pertinent dynamic analysis procedures for study of the dynamic rties and response of large free-standing towers to wind and earthquake loadings. dition, the potential advantages of using add-on damping devices to limit tower nse were investigated.

An 1100 ft. (335m) television tower in Atlanta, Georgia, served as the prototype ture for the study. The tower was modeled as a space truss with three transla- l degrees of freedom per node, and the nonlinear behavior of tension-only members ccounted for in an approximate manner. The method of substructures was chosen as fective way to deal with the large number of degrees of freedom present in the . Tower stiffness and mass associated with all but preselected master (dynamic) es of freedom were eliminated using a forward elimination substructuring prodedure. ondensed dynamic model was shown to be an efficient model for use in dynamic nse analyses of the prototype structure. Vibration properties and time-history nse to seismic loading were obtained for a variety of damping cases. The number, and distribution of dampers required to produce a significant reduction in tower nse were determined. In general, results demonstrated that dampers of the viscous were most effective when placed in the upper, more flexible part of the structure, hat the size and distribution of dampers were more important than the number of rs used to limit tower response.

**PART III–TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)***

| ITEM *(Check appropriate blocks)* | NONE | ATTACHED | PREVIOUSLY FURNISHED | TO BE FURNISHED SEPARATELY TO PROGRAM | |
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| | | | | Check (√) | Approx. Date |
| acts of Theses | | X | | | |
| cation Citations | | X | | | |
| on Scientific Collaborators | | X | | | |
| nation on Inventions | | X | | | |
| ical Description of Project and Results | | | | X | Jan. 31,1979 |
| *(specify)*<br>none | | | | | |

| al Investigator/Project Director Name *(Typed)* | 3. Principal Investigator/Project Director Signature | 4. Date |
|---|---|---|
| ry J. Goodno | | Dec. 12,1978 |

98A (5-78) Supersedes All Previous Editions

Form Approved OMB No. 99R0013

## 2.a. Abstracts of Special Problem Reports

Torres, H. P., "An Investigation of the Structural Properties of a Free-Standing Television Tower," MSCE Special Problem Report, Georgia Institute of Technology, June 1976.

## ABSTRACT


Highly stressed Civil Engineering structures such as guyed and free-standing towers, and industrial storage racks exhibit a particular need for increased energy absorption capacity. The dynamic response of these lightweight and sensitive structures may be reduced with the incorporation of structural dampers that have been developed in the past few years. The study reported herein is part of a larger project investigating the feasibility of installing specific energy absorbing devices into these lightly-damped structures at predetermined locations. The particular class of structures selected for study are free-standing television towers. In particular, the 1035 ft. (315.6 m.) Channel 17 tower in Atlanta, Georgia, will serve as the prototype structure.

The structure consists of 987 joints and 2850 members; the descriptive data for the tower (joint coordinates, member incidences, member properties, etc.) will be properly assembled and prepared in punched card form for later use with computer programs. An analytical model of the structure will be developed, taking into consideration the presence of planar joints and "tension-only" members. The tower will be modeled as an space truss with selected translational degrees of freedom. The Georgia Tech version of ICES-STRUDL and the stiffness method will be used to assemble the condensed stiffness matrix for the structure. The mass matrix for the structure will be determined by lumping tributary masses at structure degrees of freedom. The frequencies of vibration and mode shapes will be determined.

Nelson, M. G., "Substructure Analysis of Space Truss Structures with Discrete Damping Devices," MSCE Special Problem Report, Georgia Institute of Technology, December 1977.

## ABSTRACT


The need for research leading to a better understanding of structural damping is clearly evident. Highly stressed civil engineering structures such as free standing towers exhibit a particular need for increased energy absorption capacity. This study considered the feasibility of installing specific energy absorbing devices into one of these lightly damped structures at predetermined locations.

Because of the large size of the prototype structure, a practical method had to be found to assemble a reduced analytical model of the structure. The method of substructuring was selected.

The structure's damping properties were assumed to be the sum of damping due to internal molecular friction in the material and damping due to the addition of discrete dampers in the prototype structure. By varying the number, size, distribution, and arrangement of damper elements, the damper configurations that resulted in a significant reduction in the tower's dynamic response were determined. Procedures for assemblage of a structure's damping matrix for both internal and discrete element damping are presented in this report. Dynamic response of the prototype structure with varying damper arrangements is presented in a companion report.

Palsson, H., "Influence of Discrete Dampers on Seismic Response of a Free-Standing Tower," MSCE Special Problem Report, Georgia Institute of Technology, December 1977.

ABSTRACT

Energy absorption in lightly-damped free-standing towers is the subject of an NSF-sponsored research project currently underway at the Georgia Institute of Technology. The results presented herein were generated during the course of this research program.

The 1047-foot (319.3 m) Channel 17 tower in Atlanta, Georgia, was selected as the prototype structure for this study. A computer model of the tower was constructed to investigate the effect of add-on dampers on tower response to seismic loading. The tower was modeled as a space truss with three degrees of freedom per joint. The structure was divided into 21 substructures, and artificial supports were placed at the substructure boundaries. Unit displacements were specified at each level of artificial supports in turn, and a condensed stiffness matrix was constructed from the support reactions. The stiffness matrix so obtained represented a model that had two translational degrees of freedom and one rotational degree of freedom at each of the 21 levels.

The mass of the structure was lumped at the 21 levels. Very little computational effort was required to obtain the entries in the mass matrix that correspond to the translational degrees of freedom. However, no rational procedure currently existed for computing the entries in the mass matrix that correspond to the rotational degrees of freedom in an open lattice structure. Therefore, parameter studies were performed to test the sensitivity of torsional mode frequencies to changes in rotational inertia terms in the structure mass matrix.

The effect of incorporating damping devices into the condensed analytical model of the prototype structure on structure response to base excitation was investigated. The equations of motion were integrated using direct linear extrapolation with the trapezoidal rule. Finally, the size, number and distribution of dampers required to significantly reduce tower response were determined. The study showed that tower response was more dependent on the size and distribution of dampers than on the actual number of dampers used for the structure and loading considered.

Noegel, L. L., "Nonlinear Static and Dynamic Analysis of Space Truss Structures with Tension-only Members," MSCE Special Problem Report, Georgia Institute of Technology, June 1978.

ABSTRACT

Nonlinear Static and Dynamic Analyses of Space Truss Structures With Tension-Only Members

Communication towers often contain slender members designed to carry tension forces only. Linear elastic solutions, which indicate compressive forces in tension-only members, are of little significance. Nonlinear solution procedures must be used to account for the true behavior of the structure. In this study static and dynamic analyses, which accounted for the nonlinear behavior of tension-only members, were performed on selected towers.

An initial stress method was used for the nonlinear static analysis studies. The method employed iterative stress transfer and was based on a finite element formulation. Towers analyzed were loaded with torsional and translational forces applied at joints. A special purpose computer program was written to implement the initial stress method.

Nonlinear dynamic analysis was more difficult to perform since stresses in tension-only members changed with time. Direct linear extrapolation with the trapezoidal rule was used to obtain linear displacement results at each time step. Within each time step nonlinear results were calculated using the initial stress method, with linear results providing the data needed for the initial cycle. The towers considered were loaded with torsional and

translational forces applied at joints or with initial displacements.

For all towers analyzed, inclusion of the nonlinear tension-only effect increased overall structure flexibility. The extent to which flexibility increased was shown to depend upon the type of loading, the magnitude of the pre-tension force, and the number of tension-only members present in the structure.

## 2.b. Publication Citations

1. Goodno, B. J., and Palsson, H., "Substructuring for Dynamic Analysis of Free-Standing Tower Structures," Proceedings, ASCE/ICES/CEPA Specialty Conference on Computing in Civil Engineering, Atlanta, Georgia, June 27-29, 1978, pp. 575-593.

2. Palsson, H., "Influence of Discrete Dampers on Seismic Response of a Free-Standing Tower," abstract submitted in November 1978 for review for possible presentation at the AIAA/ASCE/ASME/AHS 20th Structures, Structural Dynamics and Materials Conference to be held in St. Louis, Missouri, on April 4-6, 1978 (Student Paper Competition).

## 2.c. Scientific Collaborators

1. Barry J. Goodno, Project Director and Principal Investigator
2. Hernan P. Torres, Graduate Research Assistant
3. Kenneth Gram, Graduate Research Assistant
4. Michael G. Nelson, Graduate Research Assistant
5. Hafsteinn Palsson, Graduate Research Assistant
6. Lawrence L. Noegel, Graduate Research Assistant

## 2.d. Information on Inventions

The properties of a commercially-available automobile shock absorber were used in the descriptions of the add-on viscous dampers employed in this study. While it is felt that this represents a new and interesting application for these devices, the concept was judged to be unpatentable at this time.

## 2.e. Technical Description of Project and Results

The final technical description of the research program and its findings are presented in the Final Technical Report. The report is organized by chapter as follows: (1) introduction and objectives of the research; (2) description of the prototype structures; (3) development of the substructure analytical models; (4) influence of discrete dampers on seismic response of the towers; (5) nonlinear analysis of tension-only members and their influence on tower response; (6) and summary and conclusions.

Chapters 3, 4 and 5 are largely based upon work performed by graduate research assistants (see Section 2.c. above). Results of their work are contained in four MSCE Special Problem Reports (see the Abstracts in Section 2.a. above).

E-20-668

FINAL REPORT

# ENERGY ABSORPTION IN LIGHTLY DAMPED FREESTANDING TOWERS


By

B. J. Goodno
H. Palsson



Prepared for

NATIONAL SCIENCE FOUNDATION
DIVISION OF ENGINEERING

GRANT ENG–75–10320


December 1978


## GEORGIA INSTITUTE OF TECHNOLOGY

**SCHOOL OF CIVIL ENGINEERING**
**ATLANTA, GEORGIA 30332**


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# ENERGY ABSORPTION IN LIGHTLY DAMPED FREESTANDING TOWERS

B. J. Goodno

H. Palsson

FINAL REPORT

Grant ENG-75-10320

Prepared for the

National Science Foundation

Division of Engineering

by

GEORGIA INSTITUTE OF TECHNOLOGY
School of Civil Engineering

December 1978

ACKNOWLEDGEMENT

The authors would like to acknowledge the able assistance of graduate students Hernan P. Torres, Kenneth G. Gram, Michael G. Nelson, and Lawrence L. Noegel who contributed to the successful completion of this project.

Mr. Gene Wright of WTCG Television provided structural plans and other information on one of the prototype structures.

Professor Robert D. Hanson, Department of Civil Engineering, University of Michigan, provided the data on the hysteretic behavior of commercially-available shock absorbers used in Chapter 4.

Finally, the support for this study, provided in part by the National Science Foundation through Grant ENG75-10320, is gratefully acknowledged. However, the results and conclusions presented in this report are those of the authors and do not necessarily reflect the position of the sponsor.

ABSTRACT

Dynamic analysis of large freestanding towers, which are among the most highly stressed civil engineering structures, challenges the capabilities of the structural analyst and the capacity of even modern day computers. At the same time, open-latticed steel towers usually do not have sufficient damping capacity to dissipate the energy delivered to the structures by wind and earthquake loadings. Therefore some mechanism is needed for introducing additional energy absorption capacity in order to limit dynamic response. One solution is to insert discrete damping devices into the structure at predetermined locations. These devices would be used to dissipate some of the excess energy and consequently lower the dynamic response.

To facilitate the dynamic analysis of large towers, a substructuring technique was developed which can be used for assemblage of dynamic models of self-supporting towers idealized as linearly elastic space truss structures. An overlay and condensation procedure was used to construct structure stiffness and mass matrices for preselected master degrees of freedom through forward elimination of unessential displacement coordinates. The procedure permits arbitrary selection of the number and location of dynamic degrees of freedom. Dynamic models, properties, and response to moderate earthquake ground motion are presented for 150 ft (46 m) and 1047 ft (319 m) towers to illustrate the substructure method.

Planar joints in the tower models were stabilized by adding artificial members or supports, and tension-only members were represented initially by equivalent linear tension-compression elements with small cross-sectional areas. Later, the initial stress method was used to develop a nonlinear material model, which accounted for the true behavior of tension-only members, and their influence on static and dynamic response of the towers was investigated in parameter studies.

Finally the effect of incorporating damping devices into the condensed analytical model of one of the prototype structures on structure response to base excitation was investigated. The equations of motion were integrated using direct linear extrapolation with the trapezoidal rule. The size, number and distribution of dampers required to significantly reduce tower response were determined. In general, the study showed that tower response was more dependent on the size and distribution of dampers than on the actual number of dampers used for the structure and loading considered.

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## LIST OF SYMBOLS

| Symbol | Definition |
|---|---|
| $\underset{\sim}{A}_i$ | vector of forces in substructure i or load vector at time i |
| $\underset{\sim}{A}_A$, $\underset{\sim}{A}_B$, $\underset{\sim}{A}_F$ | vectors of forces in substructure i and corresponding to A, B, and F displacement types |
| A | subscript for dependent displacements in substructure i |
| $A_x$ | cross-sectional area of member |
| $\Delta\underset{\sim}{A}_i$ | incremental load vector for step i |
| a, b | proportional damping constants |
| $\underset{\sim}{B}_j$ | balanced load vector for cycle j |
| B | subscript for temporarily-restrained displacements on the boundary between substructures i and i+1 |
| $\underset{\sim}{C}$ | structure damping matrix |
| $\underset{\sim}{C}_D$ | damping matrix for discrete dampers |
| $C_{eq}$ | equivalent viscous damping coefficient |
| $\underset{\sim}{C}_I$ | damping matrix for internal damping |
| $\underset{\sim}{C}_{FF}$ | damping matrix for master degrees of freedom |
| $\underset{\sim}{C}_M$ | add-on member damping matrix with respect to local axes |
| $\underset{\sim}{C}_{MD}$ | add-on member damping matrix with respect to global axes |
| $\underset{\sim}{D}_i$ | displacement vector for iteration cycle i |
| $\underset{\sim}{D}_i^e$ | initial estimate of $\underset{\sim}{D}_i$ |
| $\Delta\underset{\sim}{D}_i$ | incremental displacement vector for step i |
| E | elastic modulus |
| F | subscript for master degrees of freedom in the structure |
| i, j | member joint numbers or time and iteration step numbers |

| | |
|---|---|
| $k$ | substructure level number |
| $\underset{\sim}{M}_i$ | mass matrix for substructure i |
| $\underset{\sim}{M}_{AA}, \underset{\sim}{M}_{AB}, \ldots, \underset{\sim}{M}_{FF}$ | mass subarrays containing inertial force coupling terms for A, B, and F displacements in substructure i |
| $\underset{\sim}{M}^*_{FF}$ | condensed mass matrix for master degrees of freedom |
| $p_i$ | natural circular frequency for mode i |
| $\underset{\sim}{R}_T$ | member rotation transformation matrix |
| $S_{ij}$ | stiffness influence coefficient |
| $\underset{\sim}{S}_i$ | stiffness matrix for substructure i |
| $\underset{\sim}{S}_{AA}, \underset{\sim}{S}_{AB}, \ldots, \underset{\sim}{S}_{FF}$ | stiffness subarrays containing elastic force coupli terms for A, B, and F displacements in substructure |
| $\underset{\sim}{S}^*_{FF}$ | condensed stiffness matrix for master degrees of freedom |
| $\underset{\sim}{S}_o$ | initial structure stiffness matrix |
| $\underset{\sim}{T}_i, \underset{\sim}{T}_{AB}, \underset{\sim}{T}_{AF}$ | transformation matrices |
| $\underset{\sim}{U}$ | unbalanced load vector |
| $\underset{\sim}{X}_i, \ddot{\underset{\sim}{X}}_i$ | displacement and acceleration vectors for substruc i |
| $\underset{\sim}{X}_A, \underset{\sim}{X}_B, \underset{\sim}{X}_F$ | displacement vectors for A, B, and F coordinates i substructure i |
| $\ddot{\underset{\sim}{X}}_A, \ddot{\underset{\sim}{X}}_B, \ddot{\underset{\sim}{X}}_F$ | acceleration vectors for A, B, and F coordinates i substructure i |
| $\dot{\underset{\sim}{X}}_F$ | velocity vector for master degrees of freedom |
| $\underset{\sim}{X}_N$ | modal matrix normalized with respect to the mass matrix |
| $\underset{\sim}{X}_o, \dot{\underset{\sim}{X}}_o, \ddot{\underset{\sim}{X}}_o$ | initial displacement, velocity, and acceleration vectors |
| $\sigma$ , $\varepsilon$ | stress and strain for member |
| $\Delta$ | member elongation |
| $\gamma_i$ | damping ratio in mode i |

## Chapter 1
## INTRODUCTION

### 1.1 Problem Statement

Guyed and freestanding steel towers are among the most highly-stressed civil engineering structures. Typically, such towers are used in the communications industry to transmit radio and television signals over long distances, or by the power industry to support electrical transmission lines. Guyed towers have proven to be the more economical structural configuration for the taller antenna structures (guyed towers with heights of 2000 ft (610 m) have been constructed), but a large parcel of land is required for the supporting guy cables. Where space is limited and tower height requirements are moderate, freestanding (self-supporting) towers are commonly used.

The dictates or economy and efficiency in the structural design of towers have produced highly-refined tower designs with the aid of sophisticated computer programs and iterative design procedures [6,36,38,56,63].† The resulting structures are slender, lightweight, and very lightly-damped, usually less than 1% of critical [9]. The slender compression elements are susceptible to buckling failure, and the structure, as a whole, is sensitive to wind, wind combined with ice, and seismic loadings which could compromise to utility or structural integrity of the entire tower [18-21].

In spite of the up-to-date codes [2,16,17,27,47] and the sophisticated tower design computer programs incorporating the latest code provisions, several spectacular tower failures have been reported in recent years. In 1971 in Shoreview, Minnesota, a 1375 ft (419 m) triangular guyed TV tower collapsed during the final stages of construction killing seven construction workers [21]. In 1973 a 1600 ft (488 m) guyed TV-radio tower near Tampa, Florida, failed during installation of parabolic antenna [20]. Two other guyed towers collapsed in 1973 in Iowa: a 2000 ft (610 m) TV tower in Cedar Rapids [19] and a 1882 ft (574 m) tower under construction near Des Moines [18]. The latter tower was severely loaded by wind and ice which probably caused its failure.

The abbreviated list of failures reported above suggests that perhaps all factors affecting tower performance are not completely understood, and that more research on tower behavior is needed. Certainly, improved construction and inspection procedures, as well as more comprehensive evaluation and reporting of structural failures, are essential if such failures are to be eliminated.

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†Numbers in brackets refer to corresponding items in References.

Dynamic analysis of large towers for wind and seismic loadings represents a particular challenge to the structural analyst at the present time. Procedures for dynamic analysis of large towers containing hundreds of members and joints are inevitably time-consuming and expensive, and may tax the capacity of even modern day computers. The method of substructures offers a possible approach for efficient analysis of large and complex tower structures for dynamic loadings.

The sensitivity of towers to wind and earthquake loadings is at least in part due to the low damping capacity of open-latticed frameworks. The damping capacity of individual structures is difficult to predict and, in fact, may change significantly with time due to prior severe loadings which change the properties of the structure. Damping is typically assumed to depend upon some combination of internal elastic and inelastic deformation as well as friction between different structural elements. In lightweight structures, the deformations of massive structural elements cannot be relied upon to provide sufficient damping. Additional damping may be required to reduce dynamic response and control troublesome vibration problems which may ultimately result in the collapse of the entire structure. A better understanding of damping in structures would allow the structural designer to provide known levels of damping to control the transient response of structures. In particular, installing specific energy absorbing devices may be considered for lightly-damped structures such as towers. These devices would dissipate the energy caused by wind and earthquake excitations. The required number, distribution and type of damping devices depends upon characteristics of the structure, but necessary features of an energy absorbing mechanism are low cost, large capacity for energy absorption, resistance to fatigue and other forms of structural deterioration, and replaceability. A number of dampers with these characteristics have been developed and tested as noted below and they may be suitable for use in slender frameworks.

An additional complication in the analysis of towers is the presence of tension-only members, slender cable-like members frequently used in communication towers to stabilize the structure and to carry tension forces. The forces in tension-only members typically are small in comparison to forces in other members of the structure (unless a high pretensioning force is applied to these members), and the contribution of tension-only members to overall structure stiffness is usually negligible. However, tension-only members are needed to stabilize the structure and may play a role in the dynamic response of towers if their actual nonlinear behavior is considered.

In this study, a substructure analytical model was developed for the dynamic analysis of large, freestanding tower structures. The influence of add-on damping devices on tower response to seismic loading was investigated. Procedures which accounted for the true nonlinear behavior of tension-only members were formulated.

## 1.2 Previous Studies

A considerable amount of research on guyed towers and on self-supporting electrical transmission towers has been reported in the literature. Recent work on guyed towers includes that of Reichelt, et al. [56], who described an interactive computer program for the iterative, nonlinear analysis-design of guyed towers. The tower was modelled as a beam-column with rotational and translational spring supports representing the guy cables at each support level. As an example, the redesign of a 625 ft (191 m) U. S. Coast Guard tower, replacing steel guys with fiber glass cables, was presented. McCaffrey and Hartmann [45] accounted for the mass of the guys and for the effect of ambient temperature in developing linearized differential equations of motion for guyed towers. Natural frequencies and mode shapes for a 1090 ft (332 m) tower were presented. Several investigators have studied the stability of guyed towers under wind and ice loads. Goldberg and Gaunt [23] presented the governing nonlinear differential equations for overall tower stability including secondary bending effects. Tower instability was identified by locating that point on the load-displacement curve where a small increase in lateral (wind) load produced unusually high lateral displacements. Results of a static stability analysis of an 1100 ft (335 m), three-level guyed tower were presented. In another study, Williamson [73] examined the stability of "top-loaded" guyed towers with ice coatings of various thicknesses in combination with moderate wind. The critical ice thickness at which collapse is imminent was described, and results of the analysis of a 1500 ft (457 m) tower, guyed at seven levels, were presented.

Reports of recent investigations on self-supporting electrical transmission line towers are also readily available in the open literature. In 1967, the ASCE Task Committee on Tower Design synthesized the state-of-the-art and presented its Tower Design Guide [17,27] in order to establish uniform structural design criteria for steel transmission towers. Later, Wilhoite [72] compared the wind loading provisions of the Guide with other codes, such as the National Electric Safety Code (NESC) [47] and the Electronic Industries Association Standard (EIAS) [16] which pertain to design of transmission towers, and the American National Standards Institute Code (ANSI A58.1-1972) [2]. A direct interpretation of the above standards gave widely diverging results for a design example of a 100 ft (30 m) tower. Arena [4] also studied the design of transmission towers for wind loads, with particular emphasis on problems associated with the new higher voltage lines. Beck [6] explored the role of the computer in a systems approach to transmission line design. Rossow, et al. [59], and Lo, et al. [38], described a production computer program for the efficient design-analysis of space trusses in general and transmission towers in particular. Various capabilities of the program were detailed and computer run-times for several sample structures were presented to demonstrate the efficiency of the program. Natarajan [46] presented still another computer program for transmission tower analysis; a frontal solution technique was employed and results for a tower analysis were presented.

In contrast with the many investigations of guyed towers and transmission towers noted above, very little work on large freestanding communications towers has been reported in the open literature. In a series of reports, Chiu [8] and Chiu and Taoka [9-12] have presented results of a continuing analytical-experimental study of the wind-induced vibration of small freestanding latticed towers in Hawaii. In the analytical studies, the towers were idealized to be lumped-mass cantilever beams, and computer generated response to time varying wind forces (derived from available wind data and simulated wind records) was determined. In reference [10], analytical investigations of four towers, ranging in height from 150 ft to 400 ft (46 m to 122 m), are presented. In addition, the 150 ft (46 m) tower on Oahu was instrumented with anemometers and accelerometers at five levels and subjected to man-excited oscillations to determine natural periods and damping [9-11]. Measured and calculated periods compared very favorably for the first three tower modes. Analysis of ambient response records yielded damping estimates of 0.3 to 0.5% of critical in the fundamental mode. In another study, Ishizaki and Murota [31] instrumented a 22 story hotel and the 525 ft (160 m) Osaka TV Tower in Japan in order to determine the mean wind profile at the site and the response of the building to wind forces. Dynamic response measurements were not made on the tower.

A number of analytical and experimental studies of large towers, which are not entirely of open-latticed construction, have been reported, as well. These studies have focused on the character of tower response to wind forces, and have provided experimental data on measured wind speed profiles and dynamic properties and response. Shears [61] reported on wind and vibration measurements taken at the Emley Moor television tower in England. The paper briefly described the 1082 feet (330 m) high television tower which consisted of a 898 feet (274 m) tall reinforced concrete tubular shell, surmounted by a 184 feet (56 m) high aeria mast of steelwork construction. The concrete shaft was 80 feet (24.4 m) in diameter at the base and tapered exponentially to a minimum diameter of 21 feet (6.4 m) at the top. The steel aerial mast was generally of open, latticed construction. The tower was instrumented with anemometers and accelerometers located at key points in the structure. Some preliminary results of the wind and vibration analysis were presented, and a comparison of the experimental and analytical values showed that there was good agreement between these values.

Schneider and Whitman [60] reported on wind and vibration measurements performed at the Munich television tower. The tower was based on a circular reinforced concrete slab with a diameter of 131 feet (40 m) and variable thickness (6.6 feet to 17 feet or 2 m to 5.2 m), and consisted of a reinforced concrete shell of variable diameter and thickness. The shell diameter and thickness were 58.6 feet (17.9 m) and 6.6 feet (2 m) respectively, at the bottom. The diameter was gradually reduced at 0.82 feet (0.25 m) at the top of the shell at 813.4 feet (248 m). A plate with a thickness of 6.6 feet (2 m) terminated the concrete structure which reached a height of 981 feet (290 m). A staircase and three elevators were designed as a separate structure in the interior of the shell. The tower was also equipped with two multi-story cabins. The lower cabin had an overall height of 75.4 feet (23 m) and the upper cabin had a height of 68.9 feet (21 m). The upper cabin had an open and a closed observation deck and a revolving restaurant.

The wind induced response of the Canadian National Tower in Toronto was investigated by Isymnov and Brignall [32] during construction of the tower, and preliminary results were presented for the partially completed structure. The CN Tower consisted of a concrete shaft that extended to about 1480 feet (451 m) above the ground. The authors' measurements indicate that the wind induced dynamic response of the concrete shaft was almost entirely in the fundamental mode of vibration. They also found that the measured frequency of the fundamental mode agreed well with that predicted from a dynamic analysis of the partially-erected tower.

Substructures. - - A number of substructure methods for analysis of civil and aerospace structures have been presented in the literature over the last several decades. Early works by Przemieniecki [54], Hurty [30], and Clough [13] established several basic approaches to be used in the static and dynamic analysis of large structural systems. For example, various forms of the component mode method for dynamic analysis have been described and applied to the analysis of truss, plate, and frame-shear wall structures [5,14], but apparently the method is not suitable in all cases [5,64]. Rosen and Rubinstein

[58] are among those who have suggested efficiency improvements in computational procedures for substructure analysis. Weaver [68-70] used series elimination and the modified tridiagonal method [13] to develop the tier building model and a program for substructure analysis of framed structures [67]. He also used substructure methods to study behavior of plate systems [44], frames with shear walls [51], and soil-foundation-structure interaction problems [71]. Recent work has been concerned with use of finite element substructures, referred to as superelements, to model buildings and building components [24,25, 53], and ship structures [3].

Damping. - - Nelson and Grief [48] have summarized the variety of analytical formulations for damping which have been proposed, and have discussed ways of incorporating damping models in general purpose computer programs. In addition, a wide range of damping devices which may be suitable for add-on dampers in slender open-latticed towers have been considered in the literature. Mahmoodi [40] discussed the possibility of using mechanical dampers as nonload-carrying elements in tall buildings in order to reduce their amplitudes of vibratory motion. Kelly et al., [34,35,62] developed and tested several energy-dissipating devices to be used to control the stepping action of frames and towers during earthquakes. Johns, et al. [33], found that a significant increase in structural damping could be obtained if special foundation attachments with pads of elastomeric bonded cork were used to reduce the wind-excited sway vibrations of a steel chimney.

Several authors have reported on the successful use of suspended impact dampers for vibrational energy dissipation. Reed [55] discussed the use of a chain covered with a rubber sleeve and suspended with freedom to impact against a vertical channel to suppress the wind-induced oscillations of antennas, stacks, and towers. Six pendulum impact dampers were used to reduce the response of chimneys in a metallurgical plant in the town of Rustavi, U.S.S.R., as reported by Korenev [37]. Several theoretical investigations into the performance of structural and impact dampers were reported by Masri, et al., [41-43] who studied single and multi-degree of freedom systems equipped with impact dampers which employ both momentum transfer and mechanical energy dissipation during impact to attenuate the response of the primary system.

In several studies, investigators used shock-absorber mechanisms as add-on dampers to limit structure response. Hanson and Kahn [29] used commercially-available shock absorbers to temporarily control wind-induced sway of a steel chimney. Robinson and Greenbank [57] proposed a piston-like device, which dissipates energy by extruding lead back and forth through an orifice, to limit bridge vibrations. It appears likely that the force levels required for operation of the lead-extrusion absorber would preclude its use in tower structures. However, a conventional shock absorber may provide adequate levels of damping if it is properly sized for the structure and loadings under investigation.

## 1.3 Scope and Objectives

Development of an efficient linear model and pertinent dynamic analysis procedures for study of the dynamic properties and small-displacement response of wind-and-earthquake-sensitive tower structures was one of the principal goals of this research. The method of substructures was chosen as an effective way to deal with the large number of degrees of freedom present in the problem, and a series elimination technique was used to condense out all but preselected master degrees of freedom in the tower model. A variety of reduced dynamic models of two open-latticed steel towers were developed, and the influence of the modeling parameters on tower responses to selected seismic loadings was investigated. A description of the prototype tower structures selected for study is provided in Chapter 2, and the different substructure models considered are presented in Chapter 3. Vibration properties and time-history response of condensed tower models were obtained using computational procedures described in Chapter 4. Computed values were found to be in good agreement with results determined from full and approximate computer models of the structure.

An important objective of the study was to investigate the possible use of add-on damping devices to limit dynamic response of open-latticed towers. In Chapter 4, an equivalent linear, viscous dashpot model of a shock absorber is presented. The shock absorber model was incorporated in the condensed dynamic model of the overall tower, and the number size, and distribution of dampers required to attenuate the seismic response of the large prototype tower structure were determined.

The presence of planar joints and tension-only members was accounted for in an approximate manner, in the computer models of the towers. Planar joints were stabilized by addition of either artificial members or supports, and tension only members were modeled in initial studies as equivalent linear, two-way (tension, compression) members with very small cross-sectional areas. Later, the importance of considering the true nonlinear behavior of tension-only members was studied. The nonlinear model and static and dynamic response results for a segment of the large prototype structure are presented in Chapter 5.

A summary and conclusions and recommendations for further study are presented in Chapter 6.

# Chapter 2
# DESCRIPTION OF TOWERS STUDIED

## 2.1 Introduction

Several free-standing (self-supporting) communication towers were selected for detailed study in connection with the development of efficient procedures for substructure dynamic analysis of towers. The particular towers used are described below, and their substructure models are presented in Chapter 3.

## 2.2 Description of Small Tower

The 150-ft. (46-m) communication tower shown in Figure 2.2-1, whose wind-induced response was previously studied by Chiu [ 8 ], was selected as one of the test cases to be used in the study of several alternative approaches to assemblage of a reduced substructure model for towers. The tower is of triangular cross-section and has 90 joints and 306 members. It is 25 ft. (7.6 m) wide at the base and tapers at a constant slope to level 3 at a height of 143 ft. (43.6 m) above the base. From level 3 to the top, the tower has a constant width of 4 ft. (1.2 m). The tower is made up of angle shapes, and riveted connections are used throughout; specific member sizes used are contained in the paper by Chiu [8,9].

## 2.3 Description of Larger Tower

A large television and radio tower located in Atlanta, Georgia, was chosen as the primary structure for study in this research program. It is referred to as the prototype structure in this report(see Figure 2.3-1).

The structure is a free-standing, three-legged, latticed steel tower and is based on three 7 feet (2.1 m) diameter reinforced concrete caissons that extend approximately 50 feet (15.3 m) below ground surface. The caissons are reinforced with forty number 18 steel bars equally spaced. Anchor bolts of 2.5 inches (6.3 cm) diameter and 100 ksi ($6.9\text{x}10^5$ kN/$m^2$) minimum yield strength, with approximately 9 feet (2.7 m) embedded in concrete, are used to connect the tower to the foundation as shown in Figure 2.3-2.

In plan the structure is an equilateral triangle (Figure 2.3-3) and the side length of the triangle at the base is 94.167 feet (28.7 m). In elevation the structure has a constant slope of 2.55 degrees from the base to the first

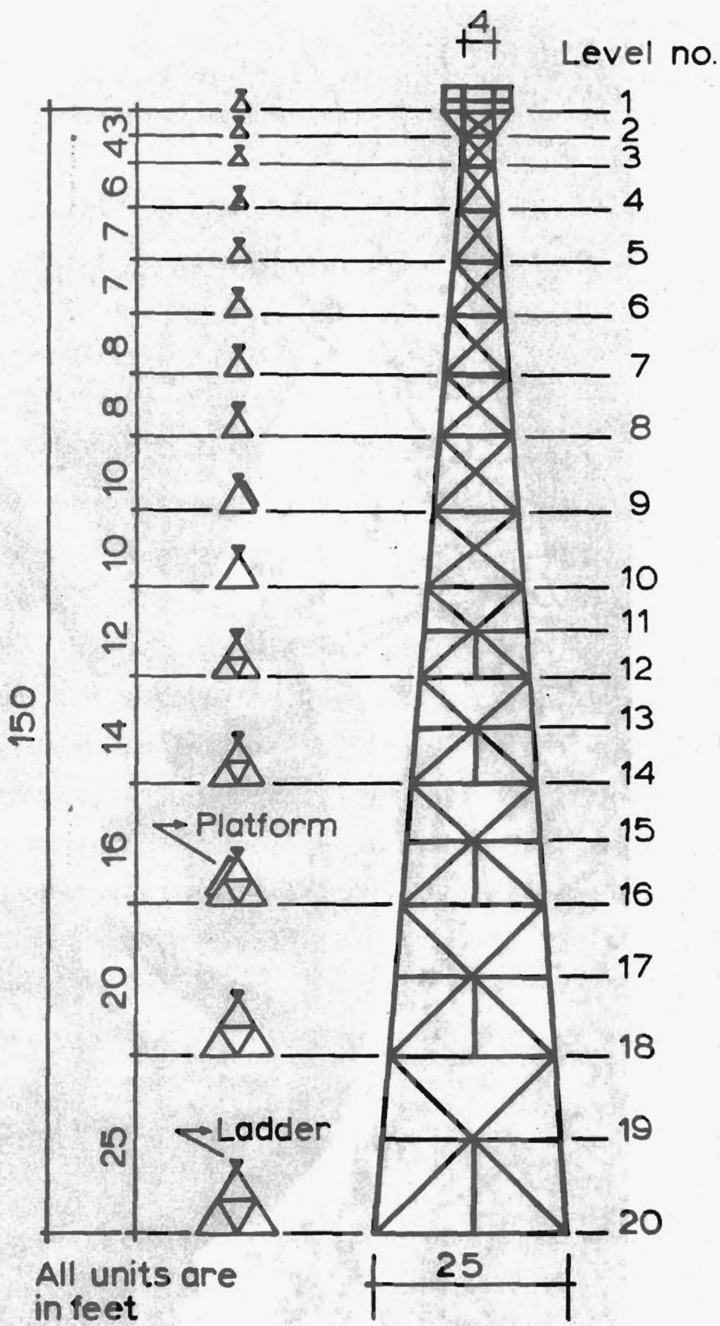



Figure 2.2-1 - Small Tower Structure

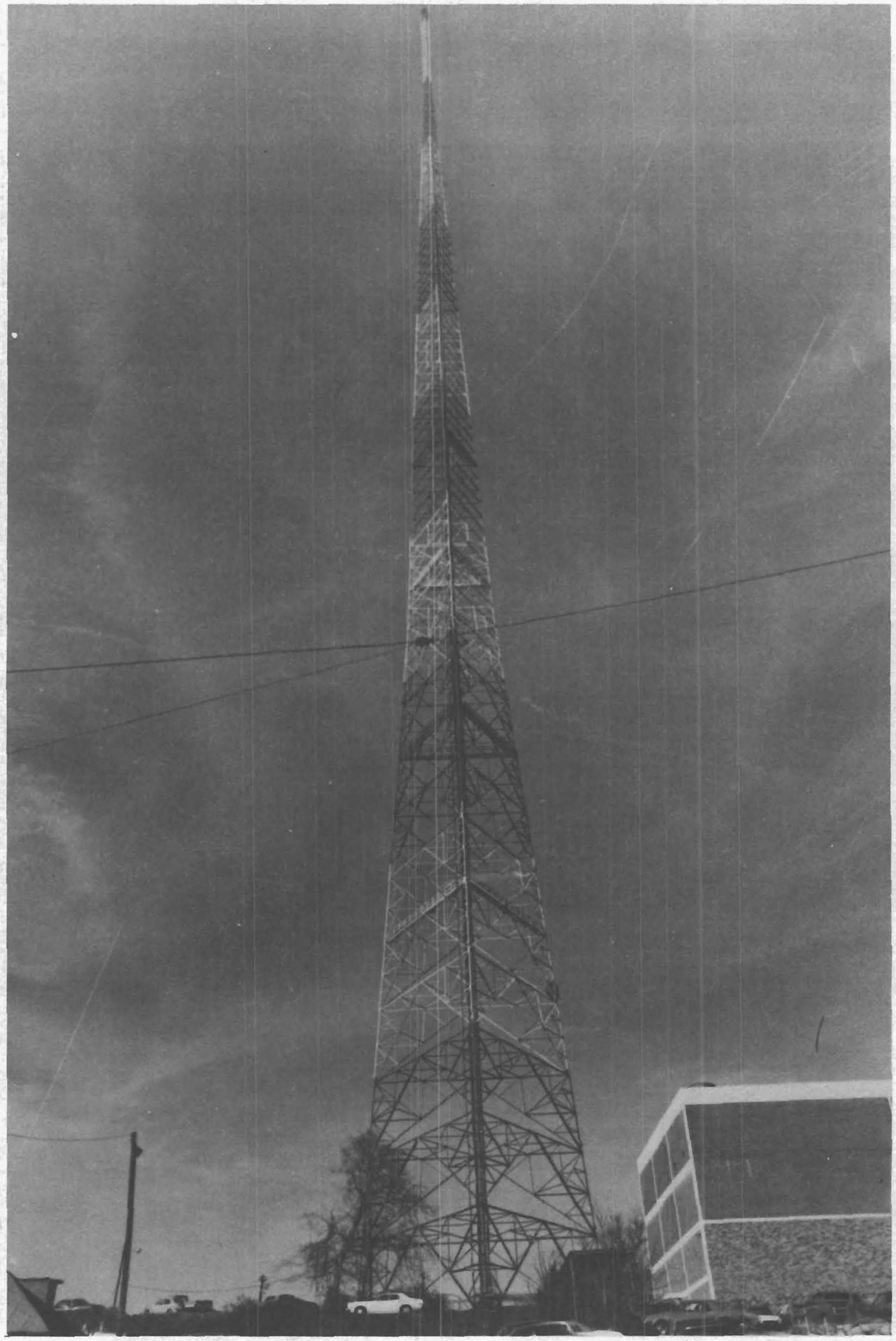


Figure 2.3-1. Large Prototype Structure

Figure 2.3-2. Foundation Detail for Large Tower

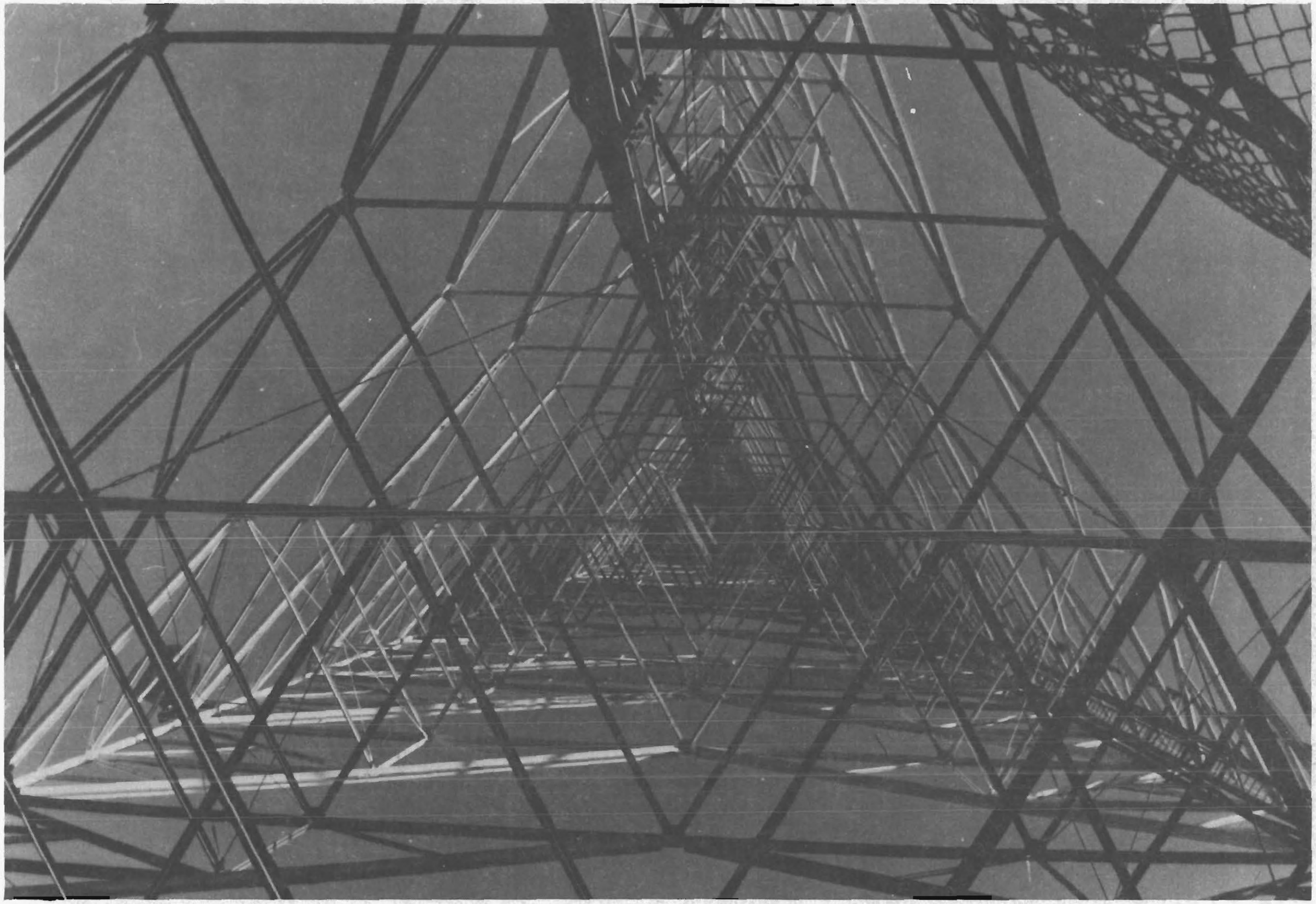


Figure 2.3-3. Tower Cross Sections

bend line at a height of 320.833 feet (97.9 m) above the base. The side length of the plan section at the first bend line is 44.594 feet (13.6 m). The tower tapers at a constant slope of 1.21 degrees from the first bend line to the second bend line at a height of 860.375 feet (262.4 m) above the base. The tower has a constant width of 5.25 feet (1.6 m) from the second bend line to the top of the supporting structure at a height of 980.375 feet (299.0 m) above the base. At this point a 66.625 feet (20.3 m) television antenna forms the uppermost segment of the tower.

The structural plans for the prototype structure indicate that most of the tower members are angle shapes of A36 steel. However, the leg members consist of solid steel bars with yield strength varying from 95 ksi ($6.55 \times 10^5$ $kN/m^2$) near the bottom to 36 ksi ($2.48 \times 10^5$ $kN/m^2$) at the top of the tower. The diagonal members are predominately angle shapes and small steel rods. The minimum yield strength varies from 36 ksi ($2.48 \times 10^5$ $kN/m^2$) to 50 ksi ($3.45 \times 10^5$ $kN/m^2$) for these members. Of particular importance are several "tension-only" members (see Figures 2.3-4 and 2.3-5)located below the first bend line. These members are solid steel bars, 5/8 inches (1.6 cm) in diameter.

The structure contains 987 joints and 2850 members, and weighs approximately 609 kips (2707 kN). Both bolted and welded connections are used in the structure (as depicted in Figure 2.3-6). Walkways are located at various levels in the structure. An access ladder and a number of transmission cables run up the southeastern leg of the tower, and the main television cable runs up the center of the tower. These walkways, ladders, and cables are assumed to add mass but no stiffness to the tower.

The structure is divided into 40 sections; figures of typical sections are presented in Appendix A and the entire tower is shown in Figure 2.3-7.

Special features. - - The stability of the prototype structure (and the small tower as well) was affected by the presence of tension-only members and planar joints if all the members meeting at that joint lie on one plane. Mathematically, the joint is unstable normal to that plane since the corresponding joint stiffness term is zero for small displacement theory. Special provision must be made for planar joints such as the addition of an artificial member or a fictitious support normal to the plane to ensure that the structure stiffness matrix is positive-definite. Procedures used to handle planar joints are discussed in Chapter 3.

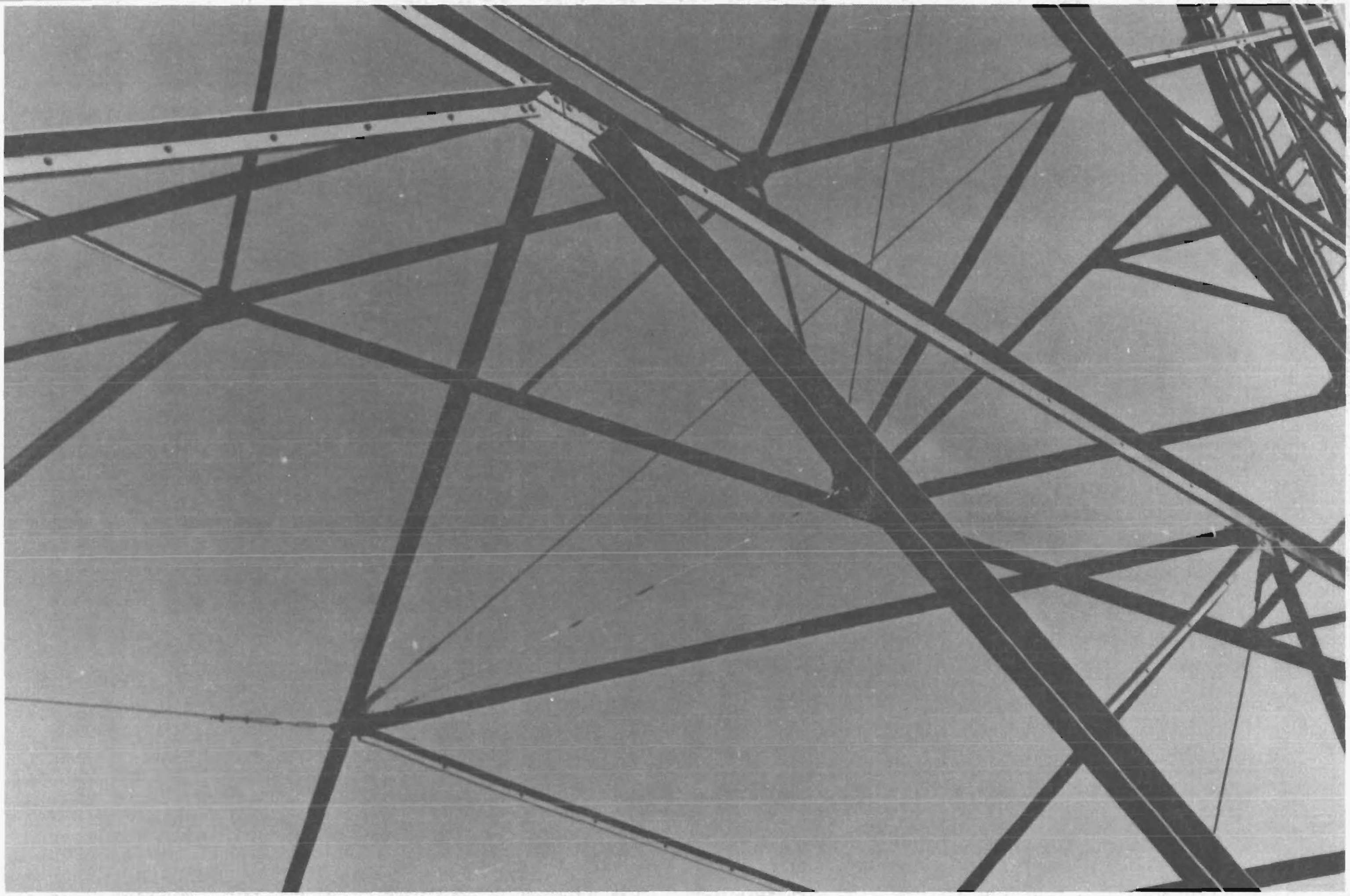


Figure 2.3-4. Tension-only Members in Leg Bracing

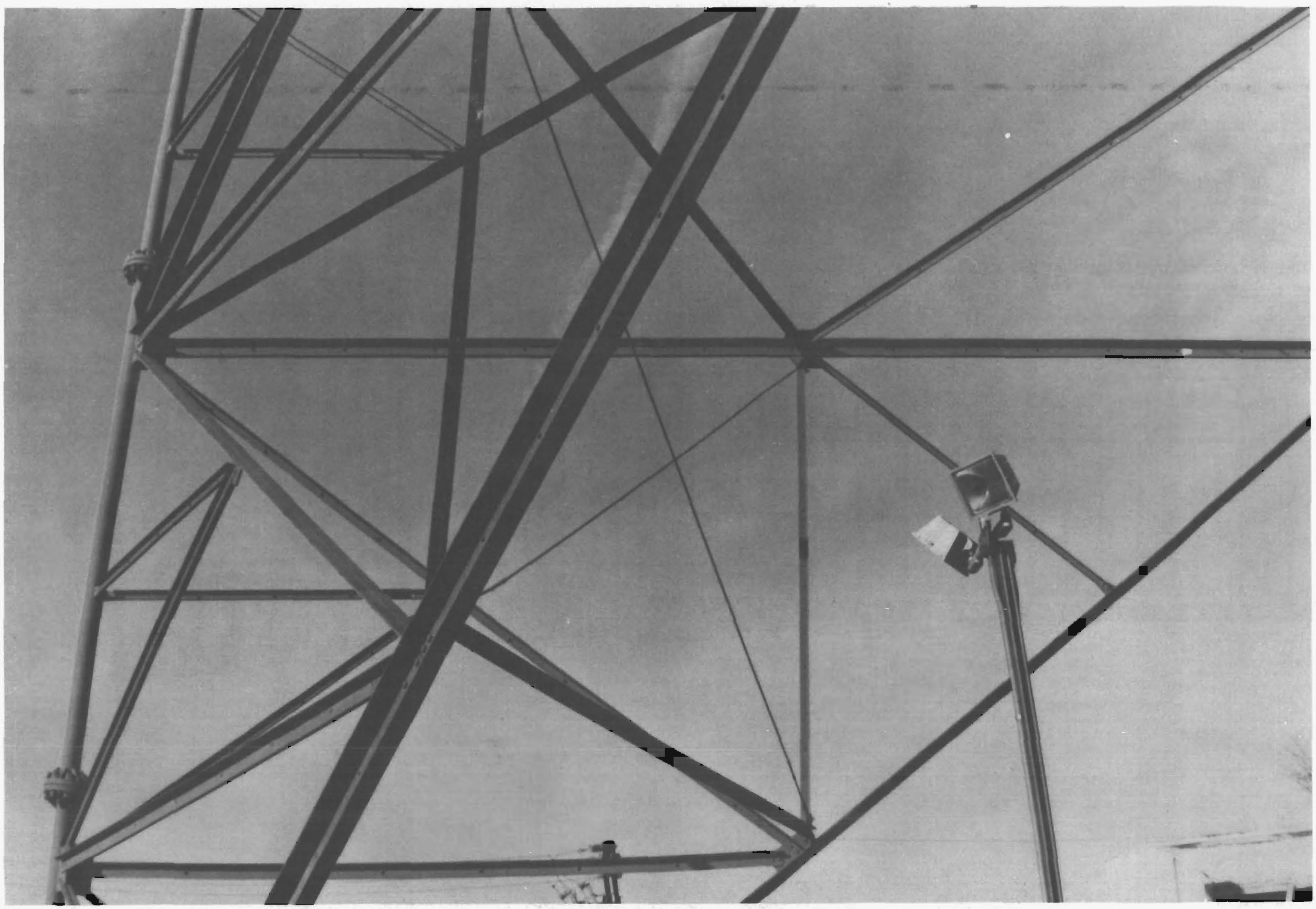


Figure 2.3-5. Tension-only Members Supporting Horizontal Bracing

Figure 2.3-6. Typical Leg and Bracing Connections

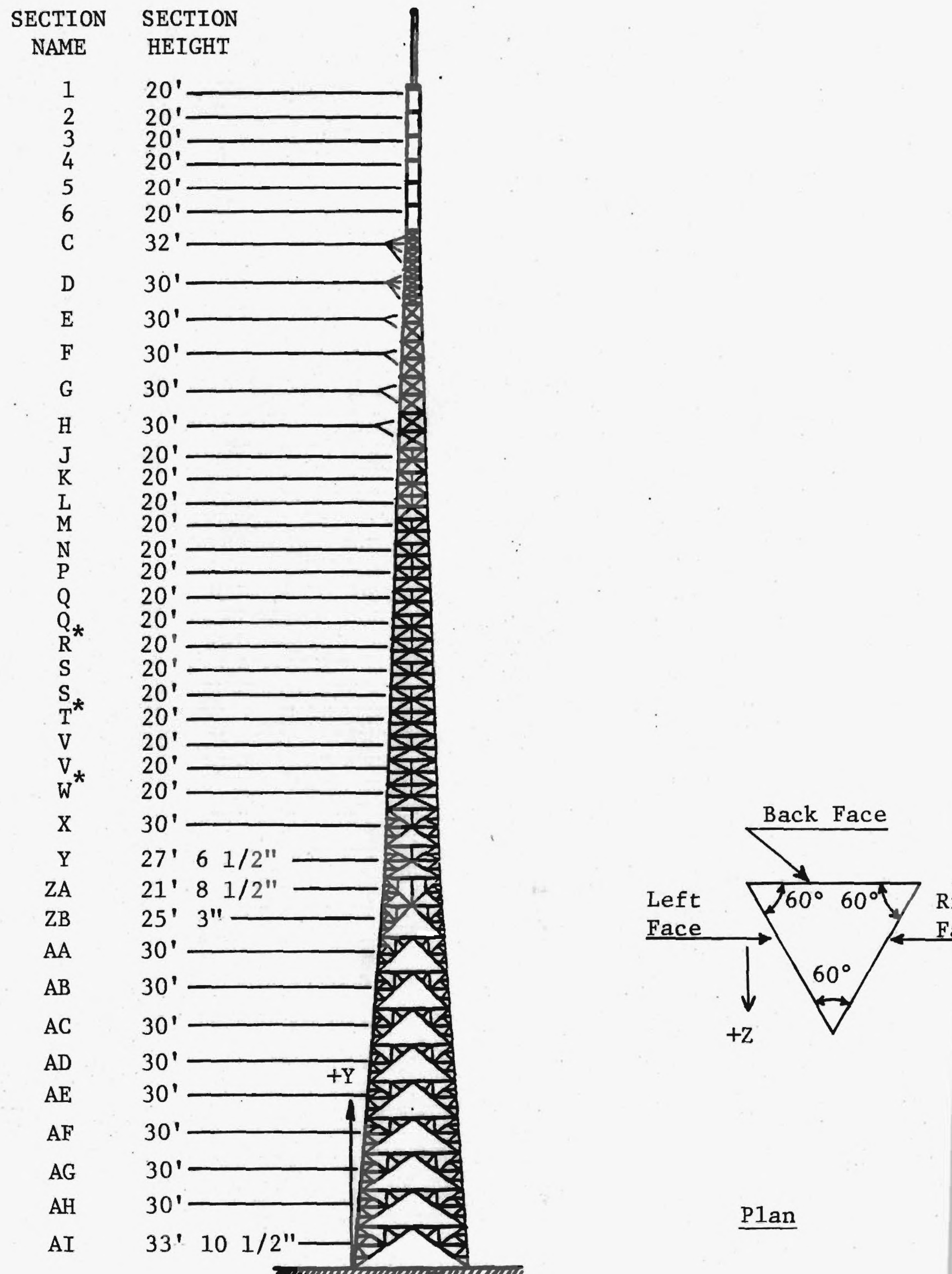



Figure 2.3-7 - Prototype Structure. Elevation and Plan View with Coordinate Axes.

The slender cable-like members incapable of carrying compressive forces were referred to as tension-only members above. Initially these members were assumed to be capable of carrying both tension and compression forces, but a preliminary analysis showed that the actual forces in the tension-only members were very small. However the tension-only members were essential to overall structural stability of the tower, but their contribution to the lateral stiffness was thought to be negligible. To more fully understand the tension-only member effect on tower response, a nonlinear model of the members was developed and was used to study the static and dynamic response of one section of the prototype structure. The results of these in-depth studies are presented in Chapter 5.

# Chapter 3

# SUBSTRUCTURE ANALYTICAL MODEL

## 3.1 Development of Analytical Model

The analytical model used to represent the two towers described in Chapter 2 was an ideal linear elastic space truss with three translational degrees of freedom per joint. The stiffness method in matrix form was the basic analytical procedure employed in the static and dynamic response analyses of the structures. With the exception of the special studies described in Chapter 5 in which the true nonlinear behavior of tension-only members in the large tower was investigated, the overall structures were assumed to be both geometrically and materially linear throughout the study.

Initially, the general purpose program GTSTRUDL [39] and direct assemblage procedures were used to develop substructure models of the large tower structure. Preliminary studies of the lowest three sections of the tower were conducted to determine the sensitivity of the structure to mass and stiffness assemblage procedures. Later, a model of the entire prototype structure was developed and used to perform frequency and dynamic response analyses. As the culmination of these structural assemblage studies, a special purpose computer program, based upon a series reduction substructuring procedure, and permitting arbitrary selection of structure dynamic degrees of freedom, was developed and used to obtain the frequencies and mode shapes of the structure.

The small tower, described in the previous chapter, was used as a test case for the series reduction substructuring scheme and computer program. Measured vibration frequencies were available for comparison with calculated values so the suitability of a variety of dynamic model assemblage procedures was investigated.

In this chapter, the substructure analytical models and assemblage procedures are discussed in Sections 3.2 and 3.3. The analytical model describes the elastic and inertial properties of the structure in the form of condensed stiffness and mass matrices for selected degrees of freedom. A variety of substructure approaches were used to determine the dynamic properties of the small tower and frequency and response results are compared in Section 3.4. Finally, frequencies and mode shapes are presented and compared for crude and refined models of the prototype structure.

Preparation of Structure Data. - - The initial step in the analysis of a structure consists of accurately defining the structure from a geometric and a structural point of view. Necessary elements for a correctly defined structure include an appropriate set of coordinate axes, overall dimensions, joint coordinates, member incidences, member properties (size, cross sectional area, strength, weight, etc.), types of connections, and support conditions.

The prototype structure contains a large number of joints (987) and members (2850), and a special purpose computer program was written to generate the joint coordinates and member incidences for the complete structure [66]. The joints and members were numbered in a spiral fashion starting at the top of the structure. This numbering scheme for the joints was selected to minimize the bandwidth of the structure stiffness matrix. As a result the number of operations and storage required in the analysis were greatly reduced.

A visual verification of the correctness of the topological data for the structure was very important due to its large size. The PLOT PLANE capability of GTSTRUDL was used to check the geometry of the structure (joint coordinates and member incidences). This command produced a line printer plot of a selected plane of the structure showing all the components in the plane, projected onto a coordinate plane. The plane to be plotted was identified by specifying only three joints or two members located on the plane. The resulting plot showed the location of all joints and members in the plane, providing a check that all coordinates and member incidences had been correctly specified in the input data.

The structure data for the small tower (90 joints, 306 members) was prepared and checked in the same manner as for the large tower, as described above.

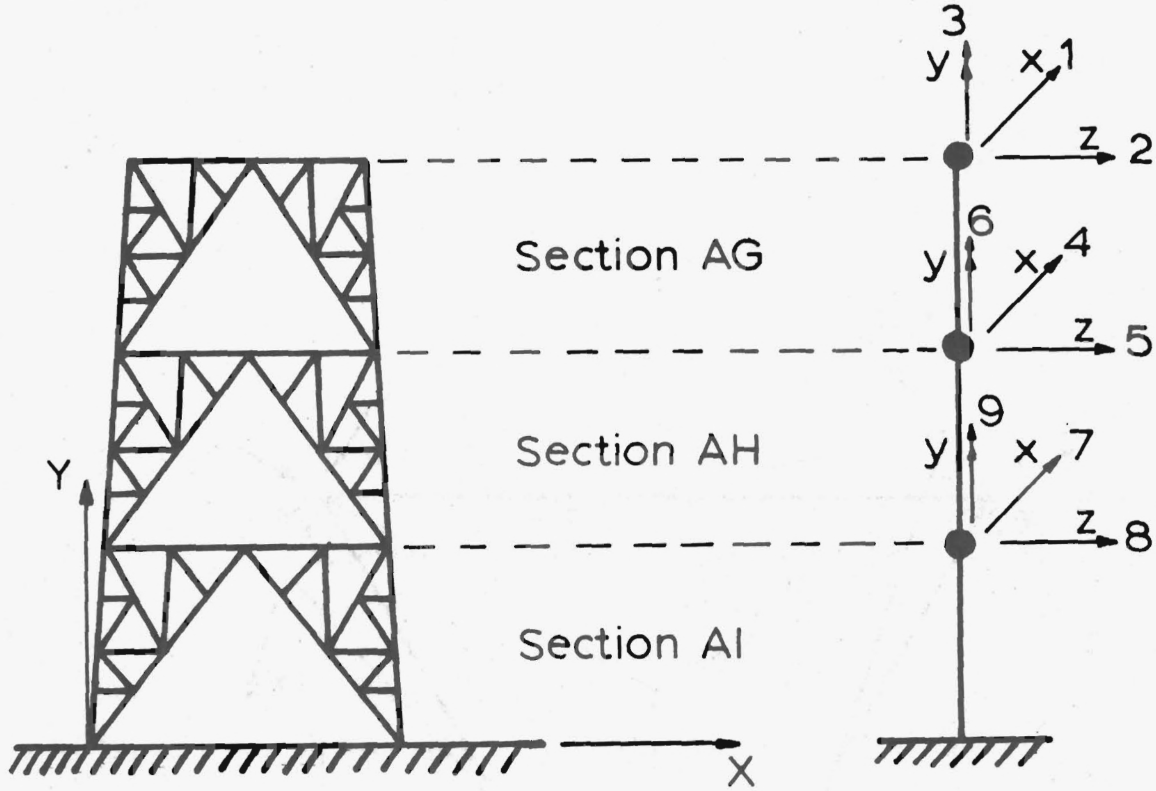
Planar Joints and Tension-only Members. - - As discussed in Chapter 2, special provision was made in development of the analytical model to stabilize planar joints to ensure that the structure stiffness matrix was positive-definite. For the initial studies of tower behavior in which GTSTRUDL was used, the JOINT RELEASES command was employed to position fictitious supports normal to the plane of the planar joints in the large and small towers. In later studies, fictitious members with small cross-sectional areas were used to stabilize planar joints.

If the nonlinear behavior of tension-only members in the large tower were to be handled in an exact manner, considerable complication would be introduced into the static and dynamic response analysis procedures. It was assumed (and later verified using the nonlinear studies described in Chapter 5) that, in so far as the analysis of the prototype structure was concerned, the true non-linear effect of tension-only members could be bracketed by two approximate linear analyses. In one analysis, the tension-only members were assumed to be fully effective in tension and compression and the actual cross-sectional areas of these members were used. In the second analysis, a more flexible structure was produced by reducing tension-only member cross-sectional areas to very small values (the areas could not be set to zero or instability would result). The true structure behavior was assumed to lie between the two extreme cases.

## 3.2 Preliminary Studies for the Large Tower

Three-Story Model. - - A portion of the prototype structure consisting of sections AG, AH and AI (see Figure 3.2-1), referred to as the three-story model, was analyzed initially so that reliable estimates of computer storage and run time could be made. In addition, the three-story model was used to study model sensitivity to mass lumping procedures and to evaluate the contribution of tension-only members to structure stiffness. The top levels of sections AG, AH and AI contained the master degrees of freedom selected from the 378 degrees of freedom available in the three-story model. Local distortions of the triangular cross section at any level were assumed to be small in comparison with overall bending and torsional motions of the structure. Therefore, structure motion was assumed to be adequately characterized by two translational and one rotational degrees of freedom at the centroid of each of the 3 levels of the model. Vertical motions were neglected all together.

The condensed stiffness matrix for the three-story model was formed by placing artificial supports at the master degrees of freedom leaving all other degrees of freedom free to displace. Unit displacements were applied at each of the master degrees of freedom in turn, and the remaining master degrees of freedom were held fixed (Figure 3.2-2). The application of a unit rotation at rotational degrees of freedom was accomplished by applying corner displacements at each level perpendicular to lines joining the centroid of the cross section and each tower leg. Structure supports at each level consisted of roller supports oriented perpendicular to the line between the leg and the centroid (see Figure 3.2-3). The stiffness influence coefficients were calculated from the reactions created at the imposed artificial supports (see Figures 3.2-2 and 3.2-3). The coefficients for translational terms were equal to the



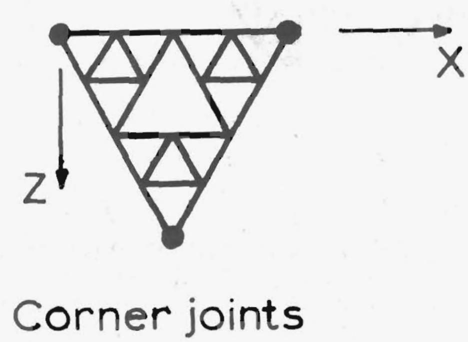




Figure 3.2-1 - Three-Story Model

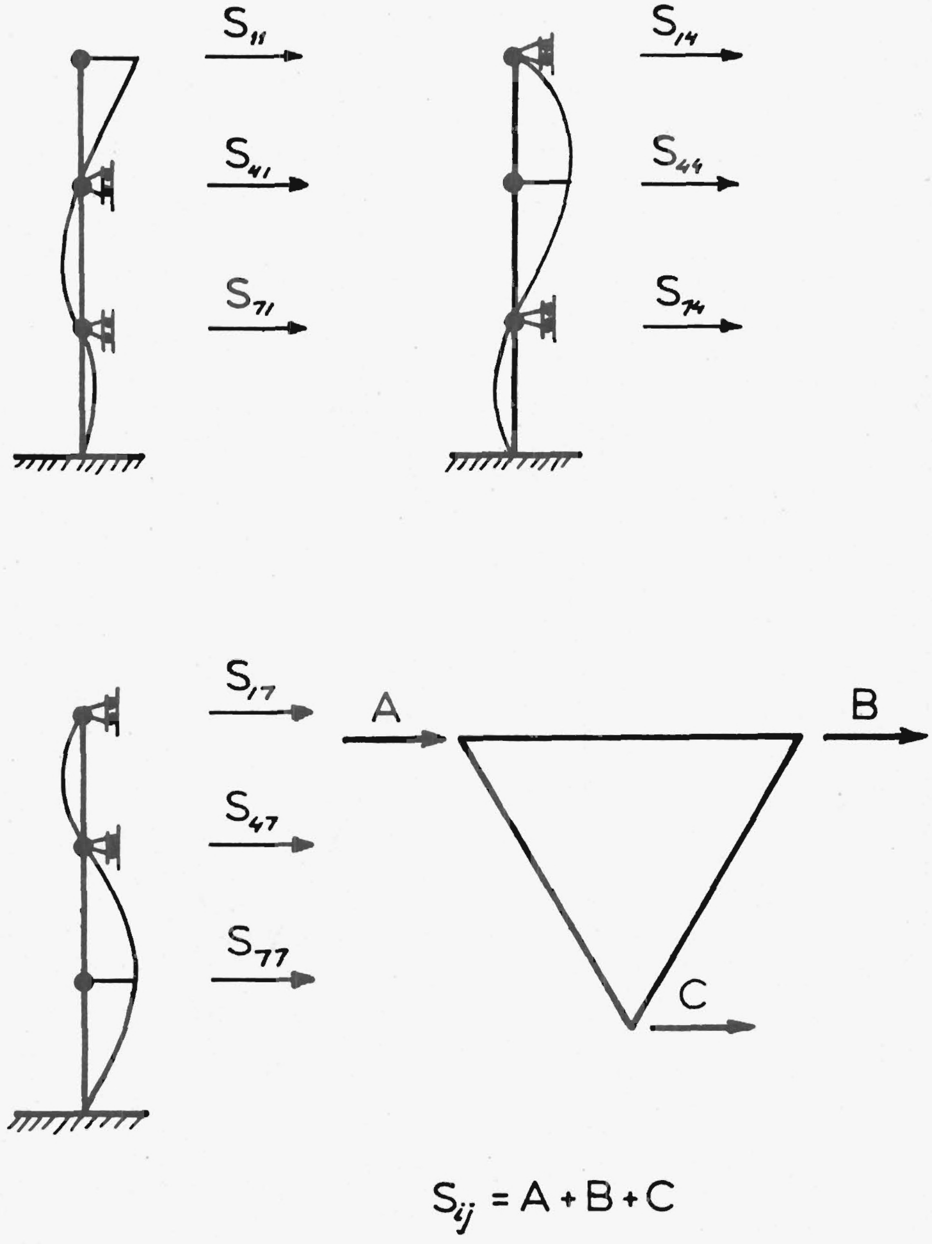



Figure 3.2-2 - Calculation of Translational Stiffness Terms

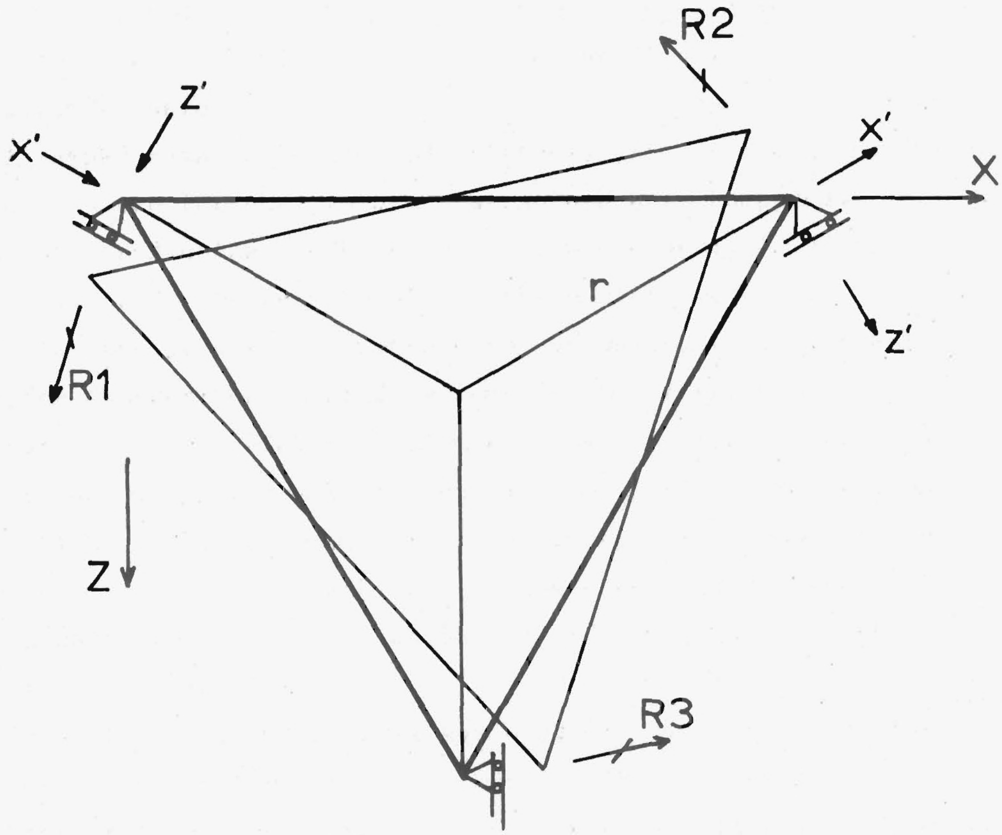



Figure 3.2-3 - Support Conditions for Calculation of Rotational Stiffness Terms

sum of the reactions at artificial supports at each level whereas those for rotational terms were equal to the sum of the moments of reactions at artificial supports about the centroid of the tower cross section.

A frequency analysis was performed to find a rational basis for lumping the mass at master translational and rotational degrees of freedom in the three-story test structure. First, the lowest 20 frequencies and mode shapes of the three-story model were obtained by using the dynamic analysis feature of GTSTRUDL. This full model analysis included all 378 degrees of freedom, three per joint. Then the appropriate lumped mass matrix for the 9 degree-of-freedom, three-story model was determined by varying the nine mass and rotational inertia terms until a good match was obtained between the lower frequencies of the two models. The same kind of comparison was performed for another three story model consisting of sections W, X and Y, located above the first bend line. As a result of these studies, it was determined that 50% of the mass of two adjacent substructures (denoted $M_L$) should be lumped at the translational degrees of freedom, and sixty percent of $M_L$ should be distributed evenly among the three corner joints at each level to compute rotational inertias.

In addition to the mass assemblage studies described above, the influence of tension-only members on lateral stiffness was investigated using the three-story model. Initially, the tension-only members were assigned their actual cross-sectional areas of 0.307 in$^2$ (2.0 cm$^2$) and an analysis for the support displacement loadings in Figure 3.2-2 performed. In this analysis, the tension-only members were treated as full tension-compression members and buckling was ignored. Member forces and reactions (i.e., here equal to the stiffness influence coefficients of the three-story model) were determined. The largest compression force in a tension-only member was 3.43 kips (15.26 kN). In a second analysis, the cross-sectional areas were reduced to 0.01 in$^2$ (0.06 cm$^2$) to approximate the loss of lateral stiffness resulting from buckling of tension-only members. While the peak compression force in the tension-only members was reduced to 1.34 kips (5.96 kN), the displacement, member force, and reaction responses of the structure were virtually unchanged. The tension-only member effect was judged to be of little significance in the computation of the condensed stiffness matrix for the three-story test structure. As a result, the original member properties were used in subsequent analyses.

As a further check on the adequacy of the two approximate linear analyses in bracketing the nonlinear effect of tension-only members, the true nonlinear behavior of the tension-only members was studied using an iterative initial stress procedure.

This work will be presented in Chapter 5.

Direct Assemblage Model for Large Tower. - - The prototype structure was subdivided into 21 sections or substructures (see Figure 3.2-4). The top level of each section contained the master degrees of freedom selected from the 2688 degrees of freedom available in the prototype structure. Based on the three-story model findings, it was assumed that the structure motion could be adequately described by two translational and one rotational degrees of freedom at the centroid of the 21 prototype structure levels. Vertical motions were neglected as before.

Using 21 x 21 lumped mass and condensed stiffness arrays, the natural frequencies and mode shapes for the prototype structure were computed. The mass matrix was constructed using the experience from the analysis of the three-story model. Based on the results of the three-story model, the entries in the mass matrix for the prototype structure corresponding to the translational degrees of freedom were calculated by lumping 50% of the mass of two adjacent substructures at each level. The lumped mass was then arbitrarily multiplied by 1.1 to account for the additional mass of cables, ladders, and walkways. The rotational inertia terms were obtained by lumping 60% of the lumped mass at the corner joints at each level. The condensed stiffness matrix was obtained using the direct assemblage procedure employed in the analysis of the three-story model.

A standard eigenvalue solution program was used to calculate the natural frequencies and mode shapes for the condensed model. The sensitivity of the lowest torsional mode frequency to changes in the rotational inertia is plotted in Figure 3.2-5. The lowest torsional mode was the 9th structure mode. Increasing the rotational inertia considerably changed the lowest torsional mode to the 7th mode. However, the fact that the lowest torsional mode was the 9th structure mode led to the conclusion that more detailed analysis of the rotational terms was of minor importance. Earthquake ground motion could excite torsional response in the prototype structure, but this effect was neglected here.

The vibration frequencies obtained from the direct assemblage model of the large tower are compared to results obtained from the general substructure model in Section 3.5 below.

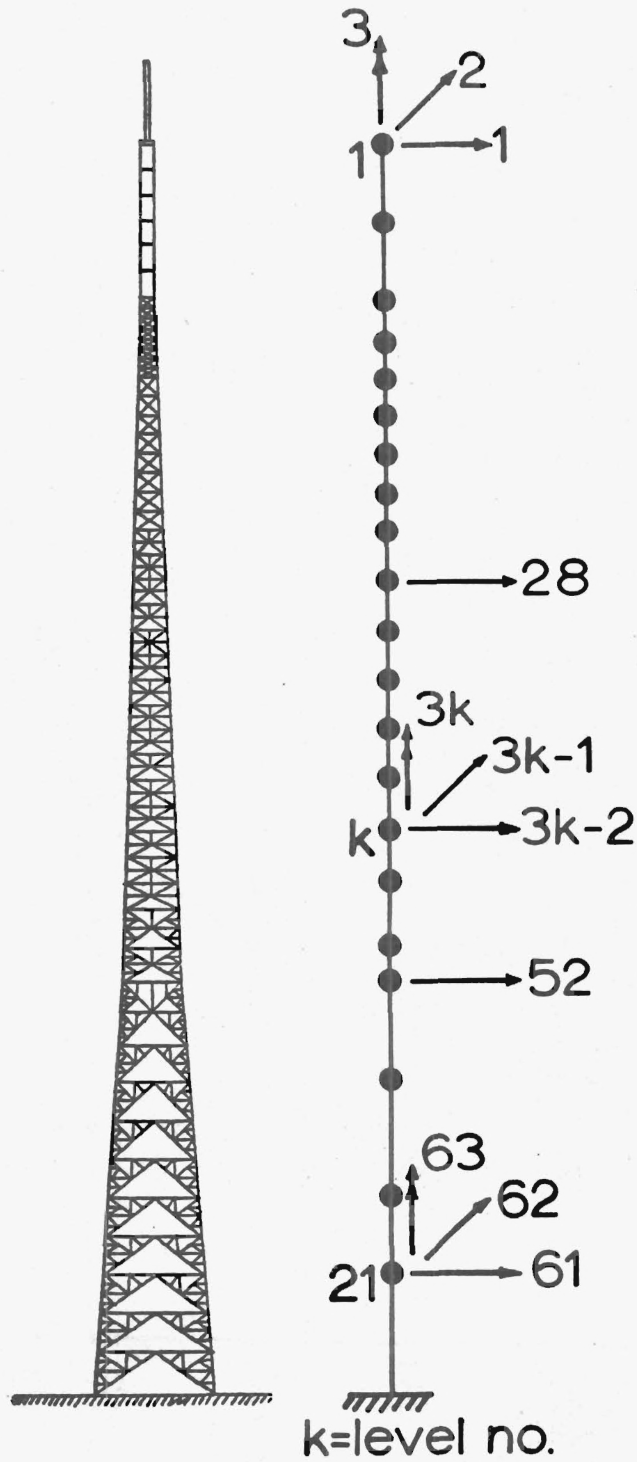



Figure 3.2-4 - Direct Assemblage Model of Prototype Structure with Dynamic Degrees of Freedom

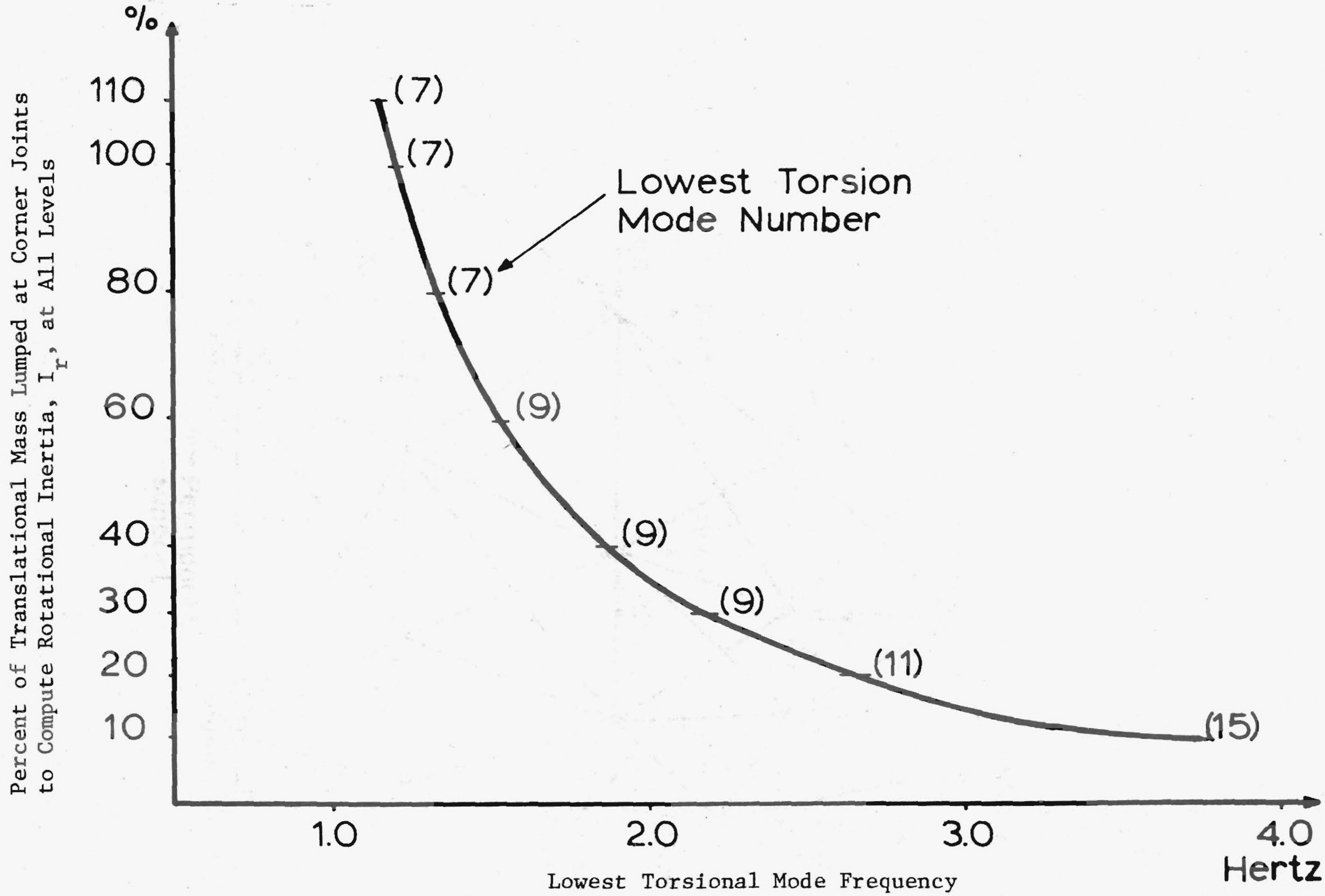



Figure 3.2-5 - Sensitivity of Lowest Torsional Mode Frequency to Change in Rotational Inertia.

## 3.3 Substructure and Alternate Methods

The direct assemblage model of the prototype structure (Figure 3.2-4) was constructed using existing software (GTSTRUDL), and was considered to be a practical approach for assembling a reduced dynamic model of a slender tower structure in which lower modes were expected to have a predominate effect on response. If a general purpose dynamic analysis program is unavailable, this may be the only alternative available for assembling a reduced model for dynamic response analysis. However, the direct assemblage procedures permitted little flexibility in selection of dynamic degrees of freedom, the lumped mass model used may or may not be an adequate representation of the inertia properties of open-latticed tower structures, and the technique for constructing the condensed stiffness array was laborious to apply. Sixty-three independent static loading conditions (three per level) were applied to obtain the support reactions (i.e., stiffness influence coefficients) at the 21 tower levels, and the support reactions were combined by hand to assemble the condensed stiffness array. This procedure required the transfer of data from one program to another and a considerable amount of card punching. A more refined tower model, permitting arbitrary selection of dynamic degrees of freedom and automated assemblage of corresponding consistent stiffness and mass arrays, was seen to offer considerable advantage. To this end, a general substructure analytical model and FORTRAN computer program were developed and applied to the dynamic analysis of the prototype structure. The general model is described below and frequency analysis results are compared for the direct assemblage and general substructure models in Sections 3.4 and 3.5.

General Series Reduction Model. - - The general substructure model used was based upon the concept of series elimination [26,27]. The tower model was subdivided into a series of substructures with no more than two substructures having a common boundary. The general tower model in Figure 3.3-1, shown as a plane truss for convenience, was represented by three substructures and temporary connection restraints were placed at boundary joints to isolate the substructures from one another. Permanent restraints, shown as R degrees of freedom in Figure 3.3-1, occur at the base of the structure.

To expedite the assembly and condensation procedures inherent in the substructure approach, it was convenient to designate the degrees of freedom

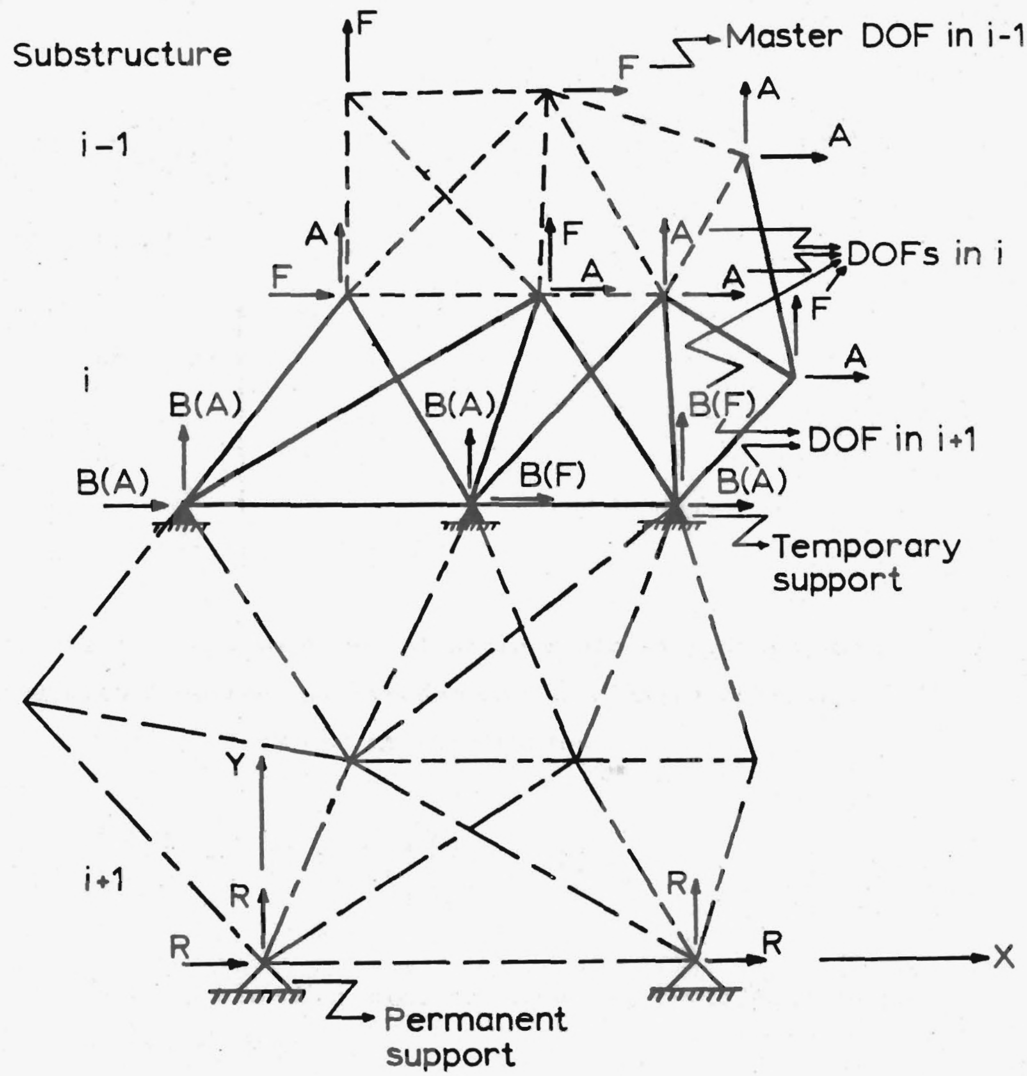



Figure 3.3-1 - General Substructure Model

in the model to be A, B, or F type displacements. Degrees of freedom of type A shown in Figure 3.3-1 were the dependent joint displacements, at free joints in substructure i, whose dynamic responses were not of immediate interest. Stiffness and mass terms associated with A degrees of freedom were eliminated from the model during a forward elimination sequence as stiffness and mass matrices for substructure i were processed. Degrees of freedom of type B were the temporarily-restrained displacement coordinates on the boundary between substructures i and i+1. B type displacements became either A or F types in substructure i+1. Finally, F degrees of freedom were master degrees of freedom in the problem which remained at the end of the structure assemblage procedure as generalized displacement coordinates for the reduced structure model. Master degrees of freedom F should be located at points of mass concentration in the tower and at other tower locations, such as antenna and equipment support points, whose motion is of primary interest.

Structural Assemblage. - - Assembly of structure stiffness and mass matrices for the reduced model followed the modified tridiagonal method [68] except that master degrees of freedom could be chosen at any joint in the model. The method involved generation of condensed structure stiffness array $\tilde{S}^{*}_{FF}$ and mass array $\tilde{M}^{*}_{FF}$ through a process of forward elimination working substructure-by-substructure from the top of the tower down to its base.

The undamped equations of motion for substructure i are

$$\tilde{M}_i \ddot{\tilde{X}}_i + \tilde{S}_i \tilde{X}_i = \tilde{A}_i \tag{3.3-1}$$

or in partitioned form:

$$\begin{bmatrix} \tilde{M}_{AA} & \tilde{M}_{AB} & \tilde{M}_{AF} \\ \tilde{M}_{BA} & \tilde{M}_{BB} & \tilde{M}_{BF} \\ \tilde{M}_{FA} & \tilde{M}_{FB} & \tilde{M}_{FF} \end{bmatrix}_i \begin{Bmatrix} \ddot{\tilde{X}}_A \\ \ddot{\tilde{X}}_B \\ \ddot{\tilde{X}}_F \end{Bmatrix}_i + \begin{bmatrix} \tilde{S}_{AA} & \tilde{S}_{AB} & \tilde{S}_{AF} \\ \tilde{S}_{BA} & \tilde{S}_{BB} & \tilde{S}_{BF} \\ \tilde{S}_{FA} & \tilde{S}_{FB} & \tilde{S}_{FF} \end{bmatrix}_i \begin{Bmatrix} \tilde{X}_A \\ \tilde{X}_B \\ \tilde{X}_F \end{Bmatrix}_i = \begin{Bmatrix} \tilde{A}_A \\ \tilde{A}_B \\ \tilde{A}_F \end{Bmatrix}_i \tag{3.3-2}$$

where $\tilde{M}_{jk}$ and $\tilde{S}_{jk}$ are mass and stiffness submatrices expressing the inertial and elastic coupling between degrees of freedom of type j and k (either A, B, or F); and $\tilde{X}_j$ , $\ddot{\tilde{X}}_j$, and $\tilde{A}_j$ are displacement, acceleration, and force vectors for displacement coordinates of type j. Dependent displacements

$\underset{\sim}{X}_{Ai}$ in substructure i were expressed in terms of boundary ($\underset{\sim}{X}_{Bi}$) and master ($\underset{\sim}{X}_{Fi}$) degrees of freedom as follows:

$$\underset{\sim}{X}_{A_i} = \underset{\sim}{T}_{AB_i}\underset{\sim}{X}_{B_i} + \underset{\sim}{T}_{AF_i}\underset{\sim}{X}_{F_i} \tag{3.3-3}$$

where

$$\underset{\sim}{T}_{AB_i} = (-\underset{\sim}{S}_{AA}^{-1}\underset{\sim}{S}_{AB})_i \tag{3.3-4a}$$

and

$$\underset{\sim}{T}_{AF_i} = (-\underset{\sim}{S}_{AA}^{-1}\underset{\sim}{S}_{AF})_i \tag{3.3-4b}$$

If matrix $\underset{\sim}{T}_i$ is defined as

$$\underset{\sim}{T}_i = \begin{bmatrix} \underset{\sim}{T}_{AB} & \underset{\sim}{T}_{AF} \\ \underset{\sim}{I}_B & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{I}_F \end{bmatrix}_i \tag{3.3-5}$$

where $\underset{\sim}{I}_B$ and $\underset{\sim}{I}_F$ are identity matrices of appropriate size, mass and stiffness terms associated with A degrees of freedom can be eliminated from the equations of motion for substructure i by the transformation

$$\underset{\sim}{T}_i' \underset{\sim}{M}_i \underset{\sim}{T}_i \begin{Bmatrix} \ddot{\underset{\sim}{X}}_B \\ \ddot{\underset{\sim}{X}}_F \end{Bmatrix} + \underset{\sim}{T}_i' \underset{\sim}{S}_i \underset{\sim}{T}_i \begin{Bmatrix} \underset{\sim}{X}_B \\ \underset{\sim}{X}_F \end{Bmatrix} = \underset{\sim}{T}_i' \underset{\sim}{A}_i \tag{3.3-6}$$

The condensed stiffness and mass arrays, for example, contained residual subarrays of the following form:

$$\underset{\sim}{S}_i^* = \underset{\sim}{T}_i' \underset{\sim}{S}_i \underset{\sim}{T}_i = \begin{bmatrix} \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{S}_{BB}^* & \underset{\sim}{S}_{BF}^* \\ \underset{\sim}{0} & \underset{\sim}{S}_{FB}^* & \underset{\sim}{S}_{FF}^* \end{bmatrix}_i \tag{3.3-7a}$$

and

$$\underset{\sim}{M}_i^* = \underset{\sim}{T}_i' \underset{\sim}{M}_i \underset{\sim}{T}_i = \begin{bmatrix} \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{M}_{BB}^* & \underset{\sim}{M}_{BF}^* \\ \underset{\sim}{0} & \underset{\sim}{M}_{FB}^* & \underset{\sim}{M}_{FF}^* \end{bmatrix}_i \tag{3.3-7b}$$

where

$$\underset{\sim}{S}^{*}_{FF_i} = \underset{\sim}{S}_{FF_i} + \underset{\sim}{S}_{FA_i} \underset{\sim}{T}_{AF_i} \qquad (3.3\text{-}7c)$$

and

$$\underset{\sim}{M}^{*}_{FF_i} = \underset{\sim}{M}_{FF_i} + \underset{\sim}{T}'_{AF_i} \underset{\sim}{M}_{AA_i} \underset{\sim}{T}_{AF_i} + \underset{\sim}{T}'_{AF_i} \underset{\sim}{M}_{AF_i} + \underset{\sim}{M}_{FA_i} \underset{\sim}{T}_{AF_i} \qquad (3.3\text{-}7d)$$

In preparation for processing substructure i+1, the temporary boundary restraints were removed in substructure i and stiffness and mass terms in the residual arrays (See equations 3.3-7a and 3.3-7b.) were shifted to new positions in $\underset{\sim}{S}_{i+1}$ and $\underset{\sim}{M}_{i+1}$. The exact position of the shifted terms in subarrays with B subscripts depended upon their new degree of freedom designations in substructure i+1 (See Figure 3.3-1.). Finally, the contributions of substructure i+1 were superimposed on the shifted residual terms from substructure i. Once the last substructure had been processed, arrays $\underset{\sim}{S}^{*}_{FF}$ and $\underset{\sim}{M}^{*}_{FF}$ remained and represented the elastic and inertial coupling, respectively, among the master degrees of freedom in the reduced tower model.

Kinematic Condensation. - - As still another alternative to the direct assemblage and series reduction approaches described above, the analyst may select a one-step reduction procedure referred to as kinematic condensation [28]. In this method, the condensation operations presented in Equations (3.3-7) are applied in one step. For large structures, the array storage and multiplications in Equations (3.3-7) cannot be handled in core, and sophisticated data management procedures (available in general purpose programs such as GTSTRUDL) must be used. While direct assemblage and kinematic condensation are not substructure methods, they represent alternate approaches which may be used to construct reduced dynamic models of large structural systems.

The direct assemblage, kinematic condensation, and substructure methods are compared in Section 3.4 for the small tower.

## 3.4 Comparison of Substructure and Alternate Methods for Small Tower

The small tower structure was divided into 19 substructures as shown in Figure 3.4-1. Planar joints were stabilized by introducing artificial members with small cross-sectional areas into the structure. Consistent mass (CM) or assembled lumped mass (ALM) was used for member elements and the mass of ladders and platforms was lumped at tributary joints.

The structure coordinate system and joint designations a, b, and c in the tower cross-section are shown in Figure 3.4-1. A variety of reduced tower models were assembled by selecting different combinations of horizontal displacements, at corner joints a, b, and c at a number of tower levels, as master degrees of freedom. The three principal substructure cases considered are listed in Table 3.4-1. Typical substructures and joint displacement types at successive stages in the elimination procedure are depicted in Figure 3.4-2 for Case 1. Also tabulated in Table 3.4-1 are full, kinematic condensation, and direct assemblage cases for comparison. In cases 4 and 5, respectively, GTSTRUDL was used to develop full and condensed models, respectively, of the prototype structure. In Case 6, GTSTRUDL and hand calculations were performed to assemble a cantilever beam model of the tower with two translational and one rotational degree of freedom at the centroid of each of 14 selected levels in the tower. For this case, stiffness matrix $\underset{\sim}{S}^*_{FF}$ was developed by inducing unit displacements in a partially-restrained structure, and matrix $\underset{\sim}{M}^*_{FF}$ was diagonal with tributary mass and rotational inertias lumped at the three master degrees of freedom per level.

Dynamic Analysis. - - The eigenvalue problem was solved to determine frequencies and mode shapes for the condensed models. The damped equations of motion

$$\underset{\sim}{M}^*_{FF}\ddot{\underset{\sim}{X}}_F + \underset{\sim}{C}_{FF}\dot{\underset{\sim}{X}}_F + \underset{\sim}{S}^*_{FF}\underset{\sim}{X}_F = \underset{\sim}{A}^*_F \tag{3.4-1}$$

were integrated using direct linear extrapolation with the trapezoidal rule to obtain the response-time histories $\underset{\sim}{X}_F$ at the master degrees of freedom. Response at other points in the structure can be determined from Equation (3.3-3) in a backsubstitution phase, if desired. Damping was taken to be a specified fraction of critical viscous damping in each mode to develop a filled damping matrix $\underset{\sim}{C}_{FF}$ for the reduced system. Finally, proportional

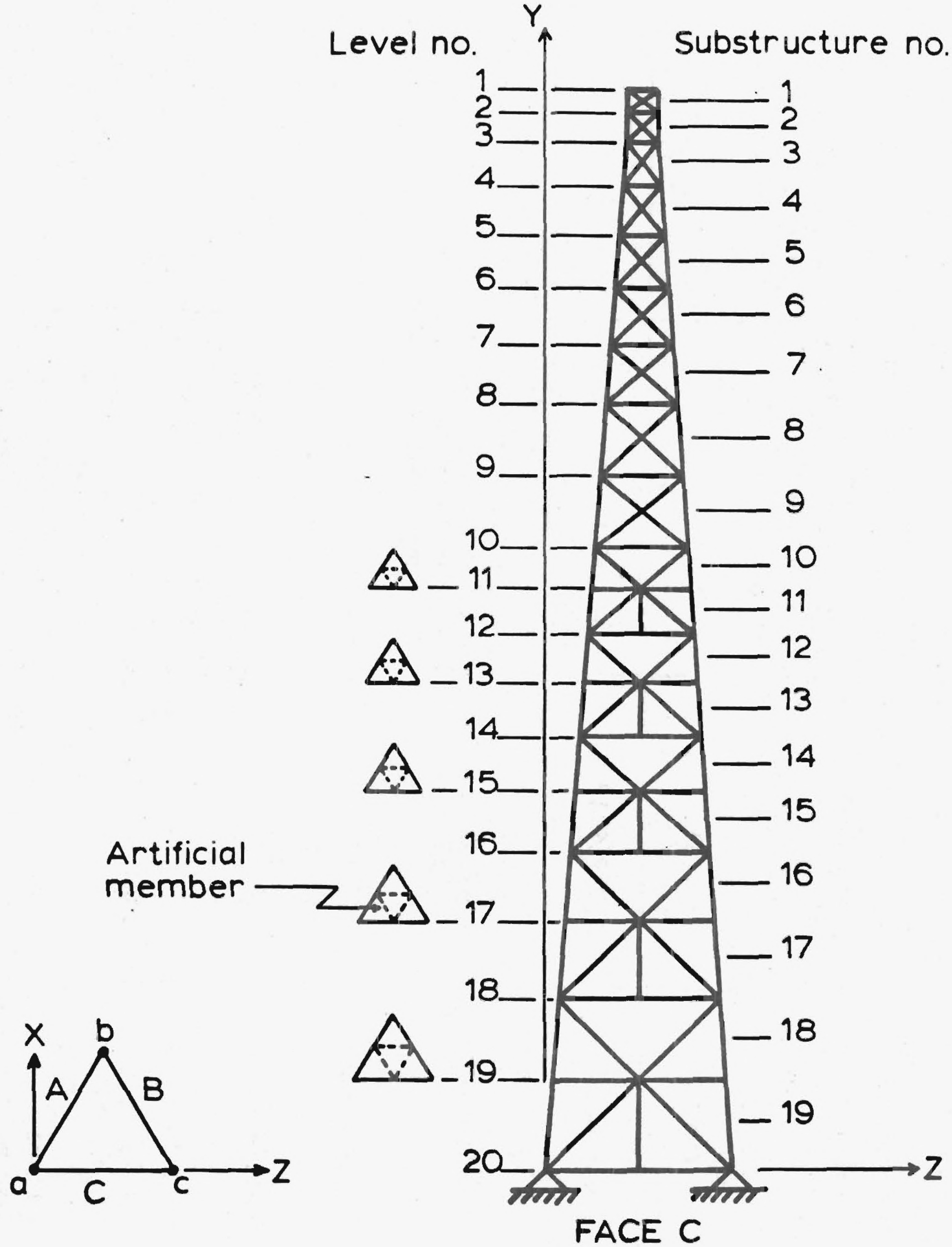



Figure 3.4-1 - Substructure Model of Small Tower

Table 3.4-1 - Model Assemblage Cases Considered for Small Tower

| Case | Formulation | Tower Levels | Master Degrees of Feeedom | |
|---|---|---|---|---|
| | | | Displacement Coordinates | Number of Degrees of Freedom |
| (1) | (2) | (3) | (4) | (5) |
| 1 | substructure | 1,3-7,9,12,16 | x and z direction translations at corner joints a,b,c | 54 |
| 2 | substructure | 1-10,12,14,16, 18 | x direction translation at corner joint b | 14 |
| 3 | substructure | 1,7,12 | same as Case 2 | 3 |
| 4 | full model | all | x,y,z direction translations at all joints | 261 |
| 5 | kinematic condensation | same as Case 1 | same as Case 1 | 54 |
| 6 | direct assemblage | same as Case 2 | x,z direction translations and y rotation at centroid of tower cross-section | 42 |

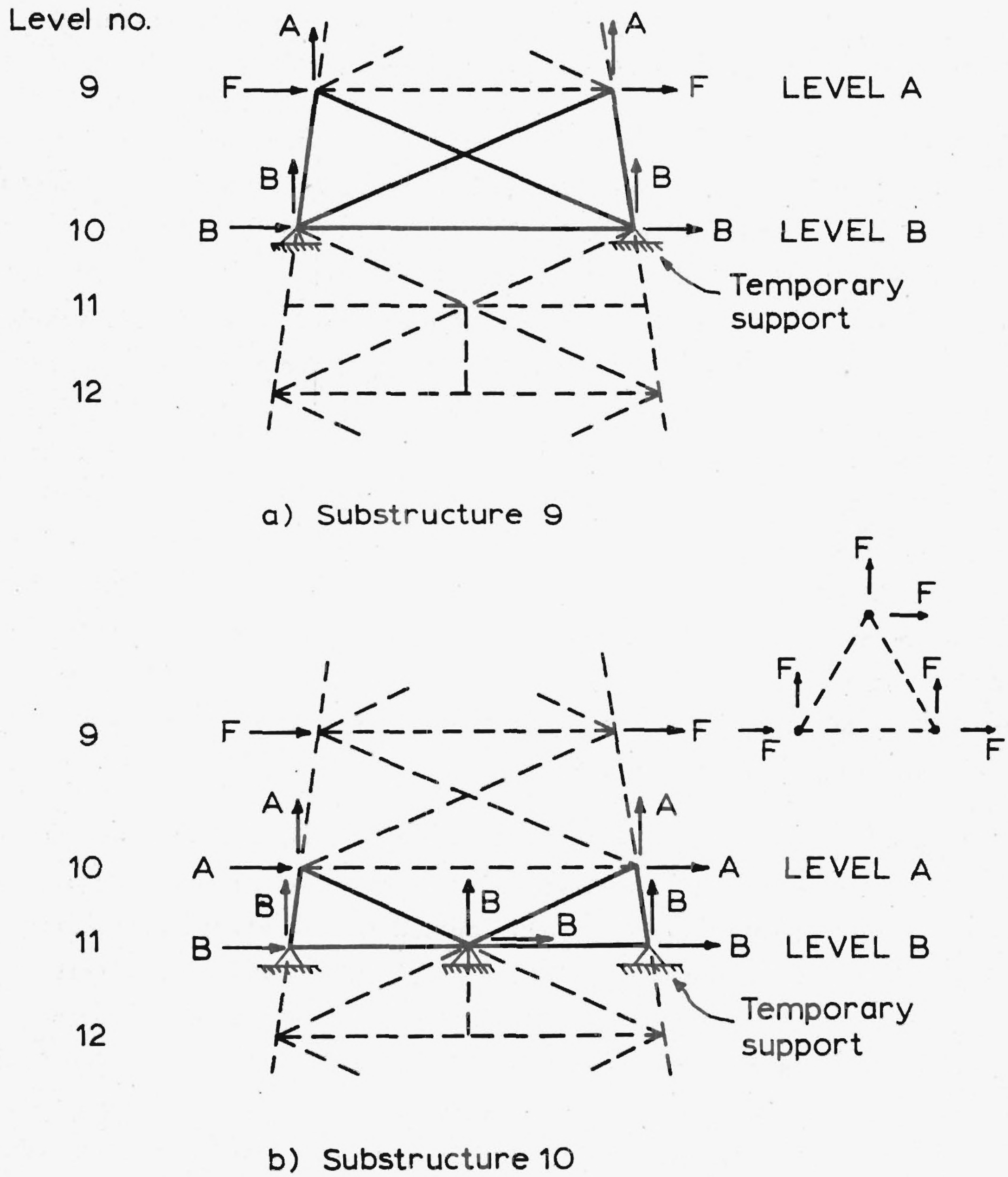



Figure 3.4-2 - Typical Substructures and A, B, and F Displacement Types for the Small Tower

loading was assumed to act on the system. Forcing functions in $\underset{\sim}{A}_F^*$ were considered to be arbitrary functions of time which could be adequately represented by piecewise constant interpolation and a small integration time step. Further details of the dynamic response analysis procedures used will be described in Chapter 4, where F subscripts and asterisk superscripts are dropped in the damped equations of motion for simplicity.

Vibration Properties. - - Vibration frequencies for consistent and lumped mass models of the tower are listed in Table 3.4-2, and are compared to previously-reported experimental values [ 9 ]. In general, substructure models yielded higher frequencies than the full and kinematic condensation cases, but results are still in good agreement with experimental frequencies.

Vibration mode shapes for the first x-direction translational and y-rotational modes are presented in Figure 3.4-3. The modes actually consist of combined translational and torsional motions and are not pure modes. Cases 1 (54 DOF substructure model), 4 (full STRUDL model), and 5 (54 DOF kinematic condensation model) are compared. Case 1 is seen to compare more favorably with the full STRUDL model mode shape than Case 5 (Figure 3.4-3a) but the opposite is true for the torsion mode in Figure 3.4-3b. In general, study of the first eight modes for Cases 1, 4, 5, and 6 (Table 3.4-1) revealed that the modal displacement shapes for all modes were in substantial agreement with full model results.

Dynamic Response. - - The dynamic analysis procedures described above were used to compute the time-history response of several models of the small tower for two dynamic loadings, each applied in the structure x direction: (1) a step-function base acceleration of 50 in/s$^2$ (127 cm/s$^2$) (Figure 3.4-4a); and (2) the N 00 E component of ground motion recorded at Ft. Tejon, California, during the 1971 San Fernando earthquake (Figure 3.4-5). These two loadings were selected because their moderate size ensured linear response, and because the resulting inertial loading on the structure depended upon the reduced mass model of the structure. The x-response at joint b at level 1 (i. e., DOF 3) to the step-function loading is presented in Figure 3.4-4b for the Case 1 model. The time-history responses for Case 1, Case 3, and Case 6 models were virtually identical; peak response values are listed in Table 3.4-3.

The responses of several reduced tower models to the Ft. Tejon record are presented in Figure 3.4-6 and peak response values are listed in Table

Table 3.4-2 - Comparison of Vibration Frequencies, in Hertz, for Small Tower

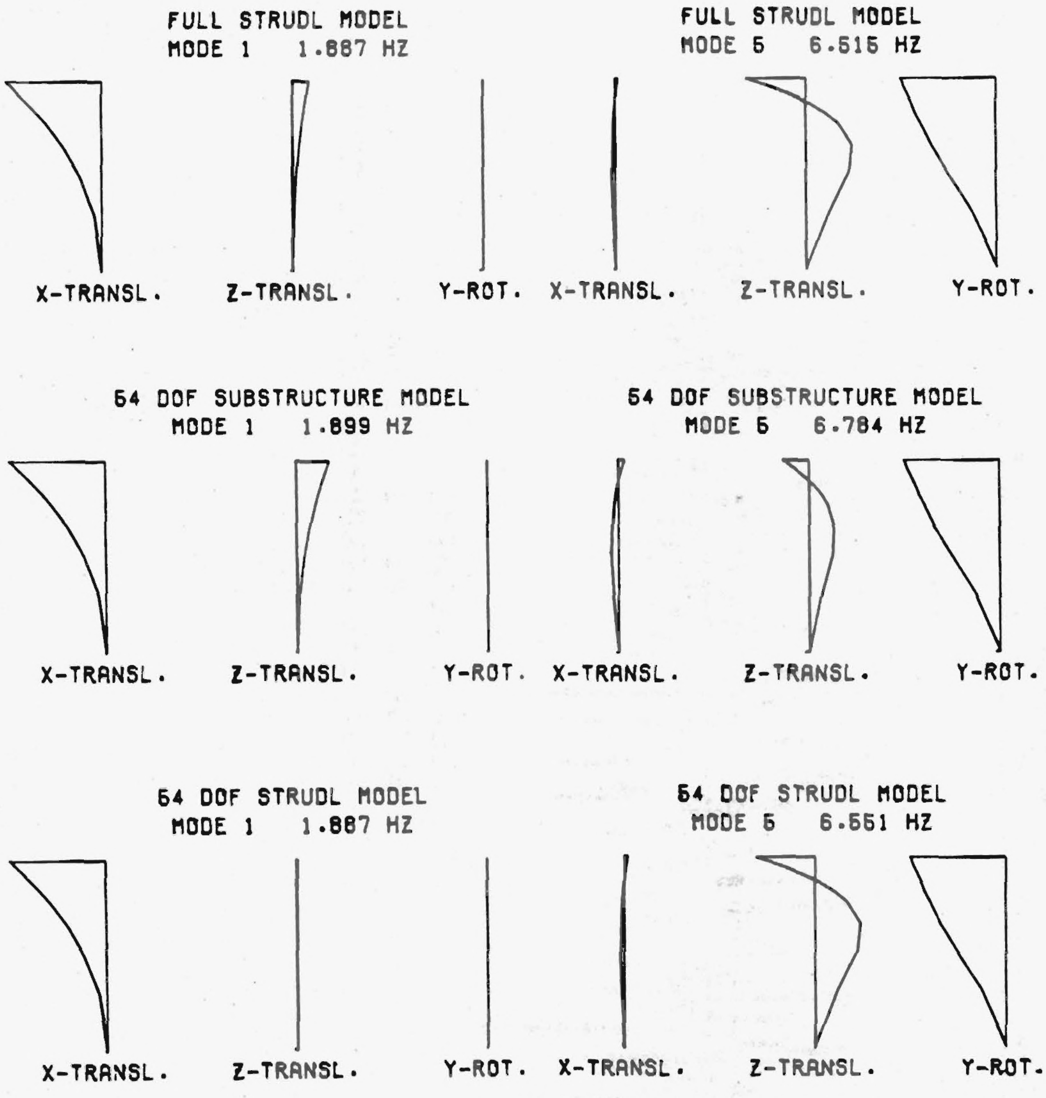
| Case | Mode | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | First X-Direction Translational Mode | | Second X-Direction Translational Mode | | Third X-Direction Translational Mode | | First Torsion Mode | |
| | CM [a] | ALM [b] | CM | ALM | CM | ALM | CM | ALM |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| 1 | 1.899 | 1.897 | 6.045 | 6.026 | 12.419 | 12.326 | 6.784 | 6.400 |
| 2 | 1.899 | 1.897 | 6.044 | 6.025 | 12.373 | 12.283 | --- | --- |
| 3 | 1.902 | 1.900 | 6.183 | 6.164 | 14.316 | 14.185 | --- | --- |
| 4 | 1.887 | 1.885 | 5.767 | 5.729 | 11.526 | 11.317 | 6.515 | 6.383 |
| 5 | 1.887 | 1.885 | 5.792 | 5.757 | 11.815 | 11.646 | 6.551 | 6.414 |
| 6 | --- | 1.899 [c] | --- | 6.042 [c] | --- | 12.229 [c] | --- | 6.344 [c] |
| Experimental [d] | 1.852 | | 5.556 | | 12.500 | | --- | |

[a] Consistent Mass Model

[b] Assembled Lumped Mass Model

[c] Diagonal Lumped Mass Model

[d] From Reference 9.




a) X-direction Translational Mode

b) Y-rotational Mode

Figure 3.4-3 - Comparison of Mode Shapes for Small Tower

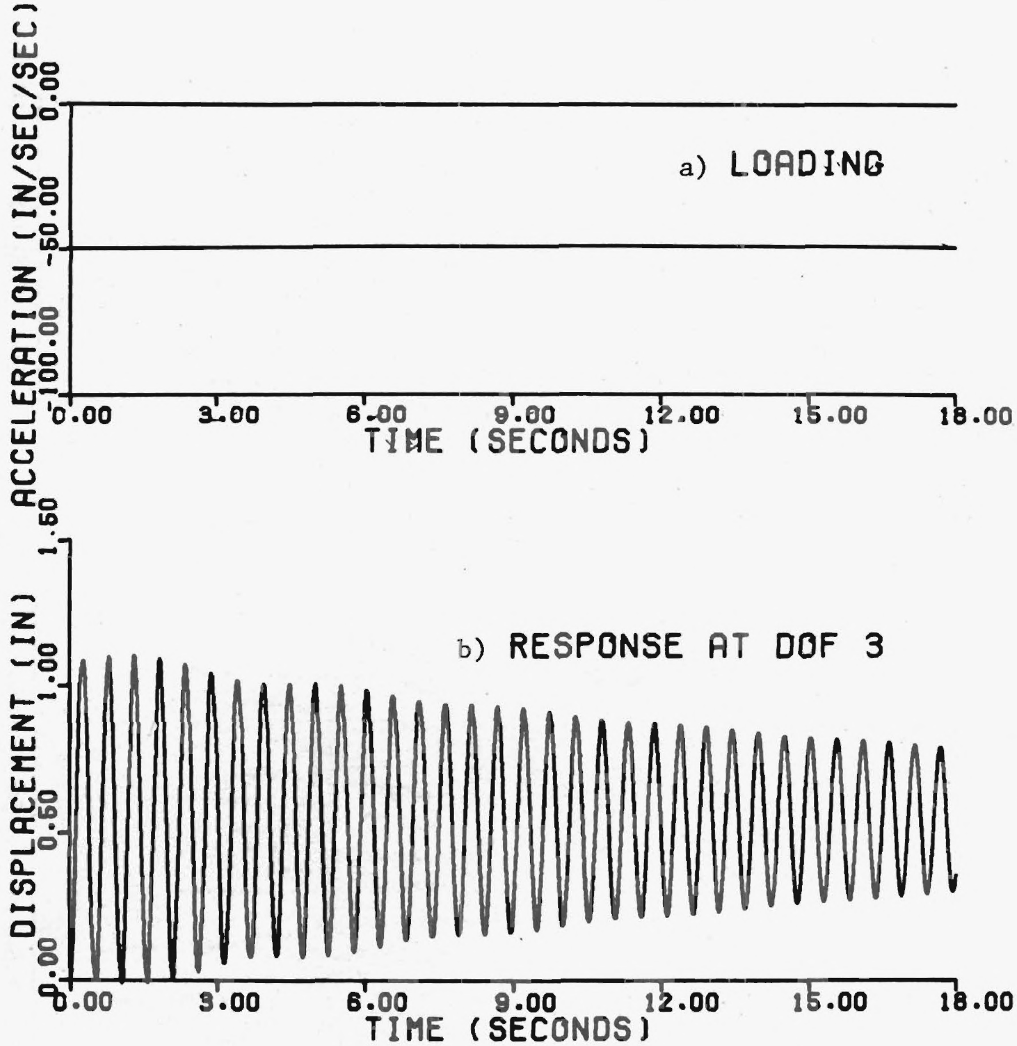



Figure 3.4-4 - Step Function Ground Acceleration and Response for 54 Degree-of-Freedom Substructure Model (1 inch = 25.4 mm)

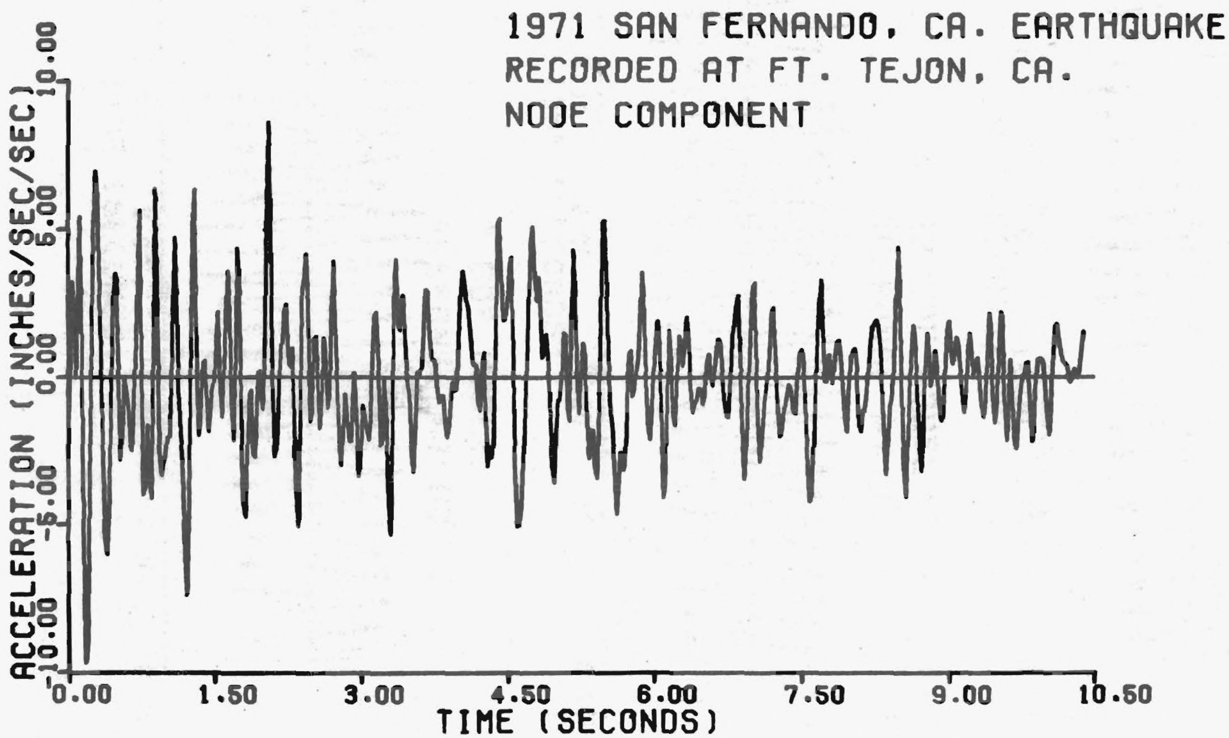



Figure 3.4-5 - Earthquake Excitation (1 inch = 25.4 mm)

Table 3.4-3 - Maximum Response at the Top of the Small Tower to Ground Acceleration Loadings

| Tower | Maximum Response in inches | |
|---|---|---|
| | Step Function Loading | Earthquake Loading |
| (1) | (2) | (3) |
| Case 1[a] | 1.104 | 0.1506 |
| Case 3 | 1.079 | 0.1664 |
| Case 5[b] | 1.032 | 0.1393 |
| Case 6 | 1.114 | 0.1522 |

[a]See Table 1.
[b]From STRUDL (Modal Analysis)
Note: 1 inch = 25.4 mm

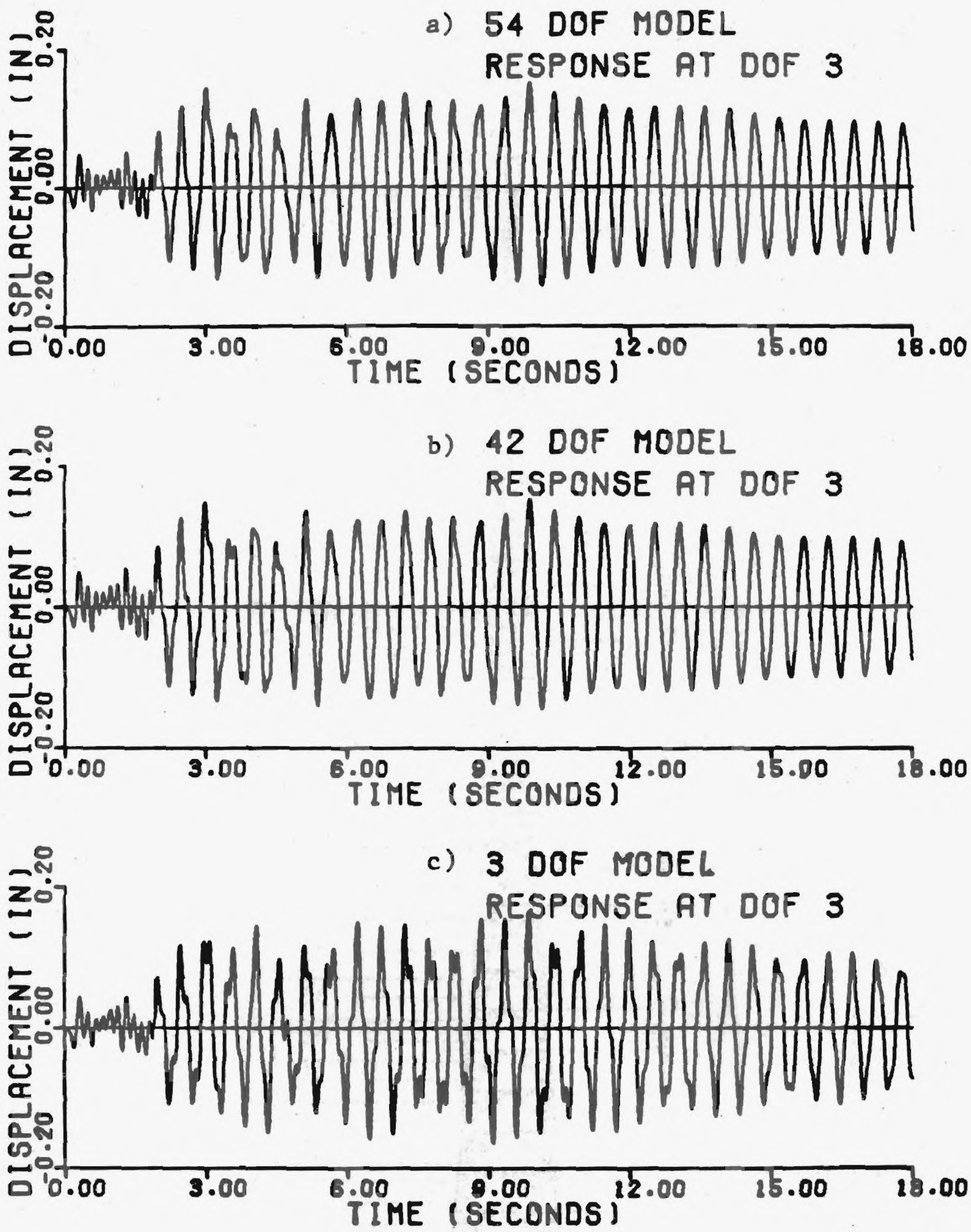



Figure 3.4-6 - Response of Small Tower to Earthquake Excitation
(1 inch = 25.4 mm)

3.4-3. The 54 (Case 1) and 42 (Case 6) degree-of-freedom models respond nearly the same, but the 3 (Case 3) degree-of-freedom model time-history shows evidence of higher mode contributions, and the peak displacement response for this case is 10% higher than Case 1. Nevertheless, the response comparison is surprisingly good considering the tower model was reduced from 261 to only 3 master degrees of freedom.

## 3.5 Substructure Model of Large Tower

Initial Tower Model. - - The large tower was divided into 34 substructures and 2 degrees-of-freedom were retained at each of 21 levels to construct a 42 degree-of-freedom dynamic model of the prototype structure. The substructures and lateral degrees-of-freedom are shown in Figure 3.5-1. Boundaries between substructures were positioned so as to limit the number of members and joints to the maximum numbers permitted by the substructure program (see Appendix B ). At present, the program dimensions permit up to 57 joints and 138 members per substructure, and up to 42 master degrees of freedom for the entire structure. These limits were governed by the available computer core storage.

The same 21 levels used in the direct assemblage model of the tower (Figure 3.2-4) were selected for placement of master degrees of freedom in the substructure model. Both consistent mass (CM) and assembled-lumped mass (ALM) were used to represent the inertia properties of the structure, and the member unit weights were arbitrarily multiplied by 1.1 to account for the additional mass of tower ladders, platforms and cabling as was done in the direct assemblage model.

Vibration frequencies for the lowest 12 modes of the direct assemblage model (Section 3.2) and the lowest 7 modes of the substructure model are compared in Table 3.5-1. The results are in good agreement. Mode shapes for the two models were essentially the same, and those for the direct assemblage model are presented in Figure 3.5-2.

It was not possible to obtain corresponding results for a kinematic condensation model of the tower due to current limitations in the dynamic analysis portion of GTSTRUDL.

Final Tower Model. - - After completion of the above work, the structural plans for the actual prototype structure became available, and it was possible to verify all member sizes and properties of attached equipment.

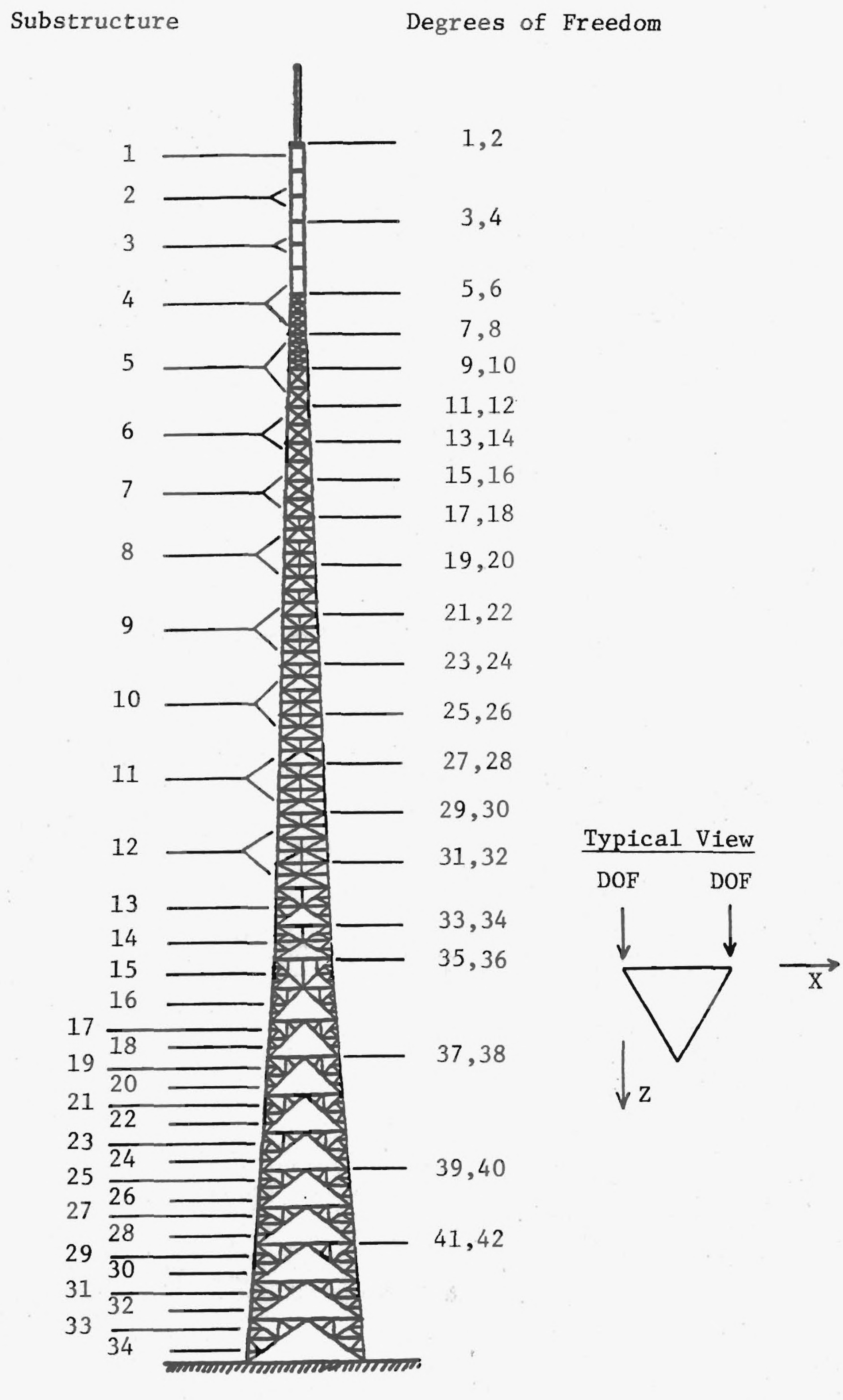



Figure 3.5-1 - 42 DOF Substructure Model of Large Tower

Table 3.5-1 - Comparison of Vibration Frequencies, in Hertz, for Direct Assemblage and Substructure Models of Large Tower

| Mode Number | Mode Description[a] | Frequency, in Hertz | |
|---|---|---|---|
| | | Model | |
| | | 63 DOF Direct Assemblage Model | 42 DOF Substructure Model[b] |
| (1) | (2) | (3) | (4) |
| 1 | X translation | 0.199 | |
| 2 | Z translation | 0.199 | 0.199 |
| 3 | X translation | 0.468 | |
| 4 | Z translation | 0.468 | 0.475 |
| 5 | X translation | 0.882 | |
| 6 | Z translation | 0.883 | 0.922 |
| 7 | X translation | 1.451 | |
| 8 | Z translation | 1.452 | 1.502 |
| 9 | Y rotation | 1.523 | 1.410 |
| 10 | X translation | 2.185 | |
| 11 | Z translation | 2.187 | 2.282 |
| 12 | Y rotation | 2.364 | 2.216 |

[a]See Figure 3.5-2 for mode shapes.
[b]Consistent mass model.

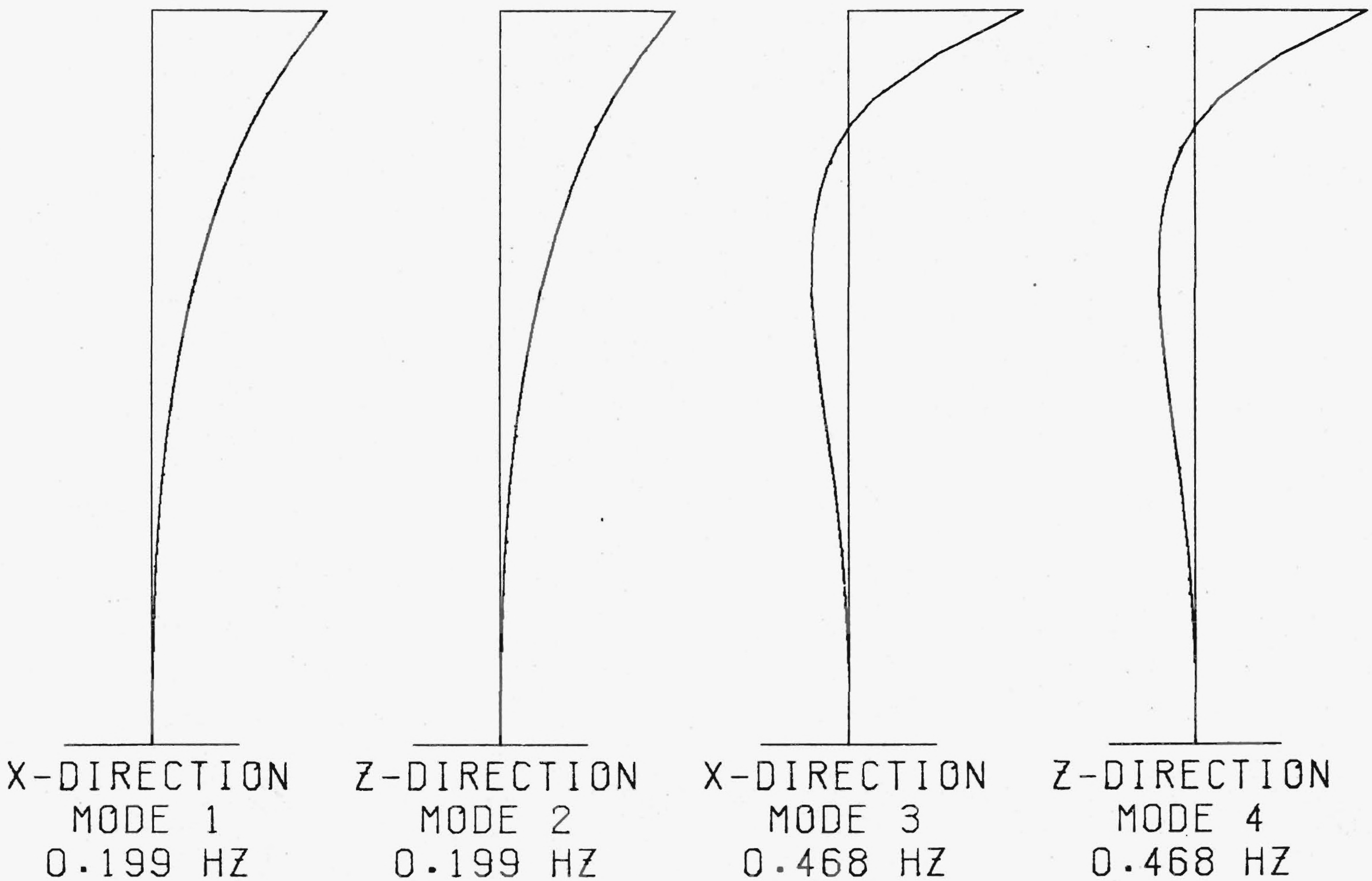



Figure 3.5-2 - Mode Shapes for the Direct Assemblage Model of the Large Tower

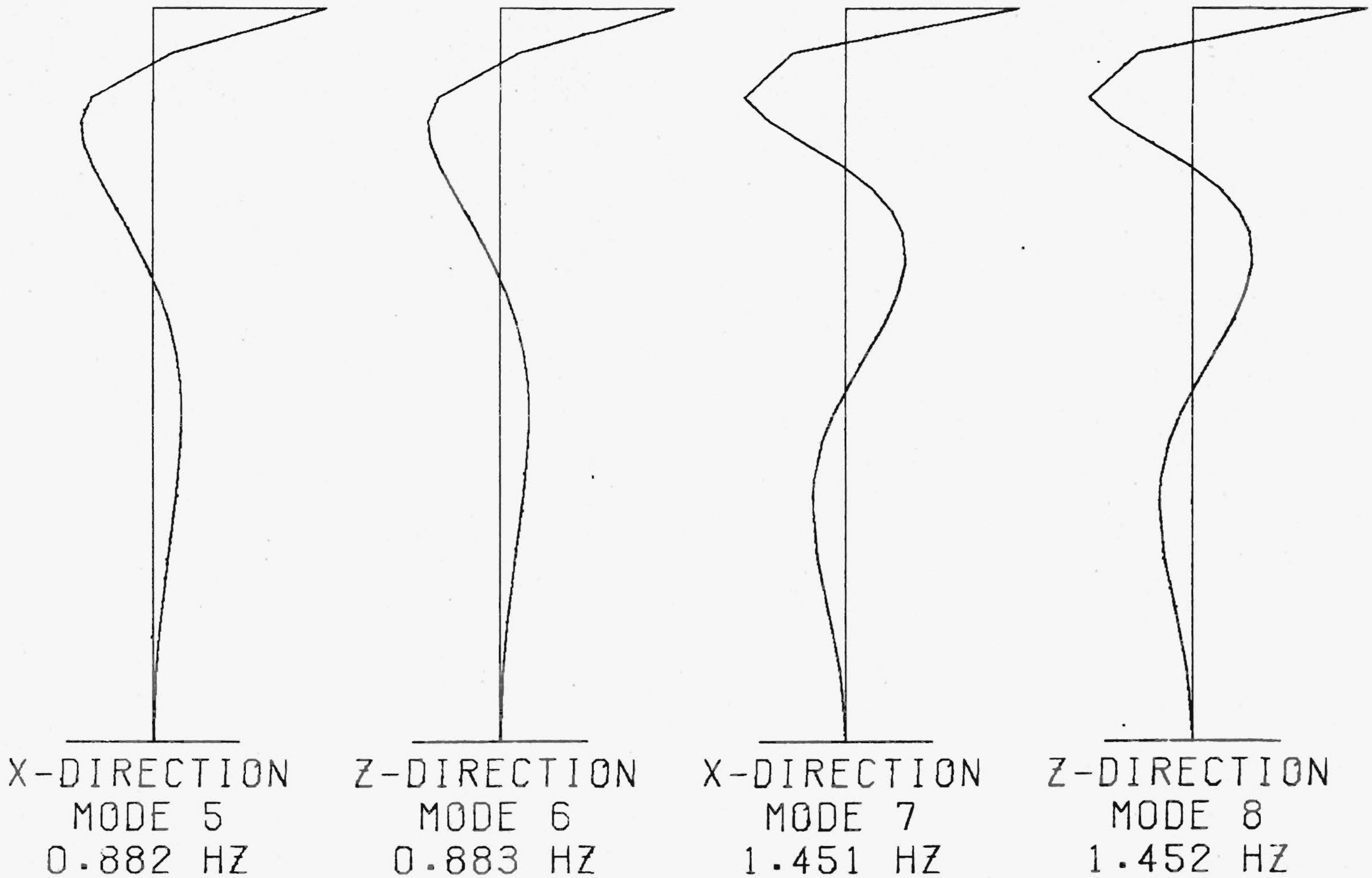



Figure 3.5-2 - Mode Shapes for the Direct Assemblage Model of the Large Tower (continued)

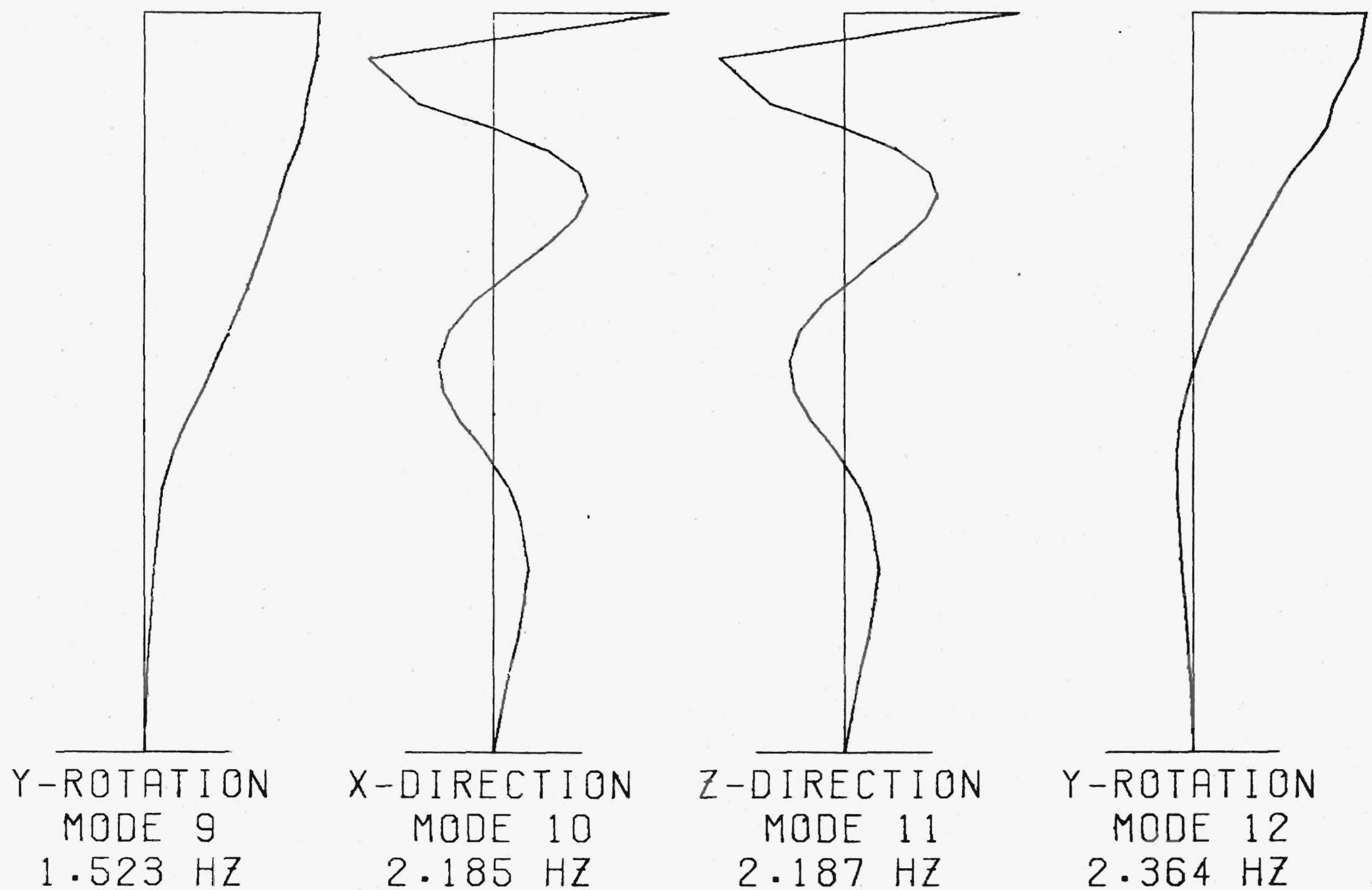



Figure 3.5-2 - Mode Shapes for the Direct Assemblage Model of the Large Tower (continued)

The original data was revised and the added mass of cables, antennas, ladders, and platforms were accounted for in the consistent mass model (i.e., in place of an arbitrary 10% increase in structure unit weight). The computed frequencies for the actual prototype structure, obtained from the revised 42 DOF substructure model, are presented in Table 3.5-2.

Table 3.5-2 - Vibration Frequencies, in Hertz, for Actual Prototype Structure

| Mode Number<br>(1) | Mode Description<br>(2) | Frequency, in Hertz<br>(3) |
|---|---|---|
| 1 | 1st Z translation | 0.174 |
| 2 | 2nd Z translation | 0.407 |
| 3 | 3rd Z translation | 0.940 |
| 4 | 1st Y rotation | 0.984 |
| 5 | 2nd Y rotation | 1.380 |
| 6 | 4th Z translation | 1.570 |
| 7 | 3rd Y rotation | 1.849 |

Chapter 4

# INFLUENCE OF DISCRETE DAMPERS ON TOWER RESPONSE

## 4.1 Introduction

Engineers do not have a good understanding of damping in structures. What is known is qualitative in nature and very little quantitative information is available, although continued study is providing additional information. The effect of damping is known to be small for response during short duration excitation. Thus damping has little effect on the dynamic response of a structure subject to a blast wave type excitation. In general it is known that the influence of damping is small for steady state response to periodic excitation when the exciting frequency is not near resonance. However, the effect of damping is of primary importance for periodic excitation at or near resonance [65]. Research indicates that in most civil engineering structures the damping ratio due to internal effects has a maximum value of approximately 15% to 20% of critical for response in the linear range.

Measurements have revealed that damping levels are considerably lower in slender frameworks such as towers. Values as low as a fraction of 1% of critical have been obtained [ 9 ], and a maximum of 5% is to be expected [74]. Some mechanism is needed for introducing additional damping into tower structures to control response and eliminate troublesome vibration problems.

The use of add-on energy-absorbing devices to limit wind and earthquake-induced response is a possible solution. Desirable features of such a device are low cost, durability, and replaceability. In this investigation, an analytical model for an add-on damper was developed, and the number, size, and distribution of devices required to attenuate the seismic response of the large prototype tower were determined [49,52].

## 4.2 Damper Model

In general, all civil engineering structures possess damping to some degree due to some unknown combination of internal elastic and inelastic member deformation together with friction between various structural elements. This internal damping can be represented by a combination of two different forms of equivalent damping, namely, viscous or velocity dependent damping and Coulomb or dry friction damping. However, viscous damping is the simplest to deal with mathematically. For this reason, resistive forces of a complicated nature are very often replaced for analysis purposes by equivalent viscous damping, expressed in the form of damping constant $C_{eq}$. The equivalent viscous damping constant is determined in such a manner as to approximate the dissipation of energy per cycle produced by the actual resistive forces. This methodology was adopted for use in this study, and the effect of other forms of damping was ignored.

The internal damping present in the structure was represented by a damping matrix $\underset{\sim}{C}_I$, and the effects of add-on dampers by a matrix $\underset{\sim}{C}_D$. The total damping in the structure was then defined by a matrix $\underset{\sim}{C}_T$ as

$$\underset{\sim}{C}_T = \underset{\sim}{C}_I + \underset{\sim}{C}_D \qquad (4.2\text{-}1)$$

Internal Damping. - - Matrix $\underset{\sim}{C}_I$ was assembled using either a simple modal or a proportional damping formulation. In simple modal damping, damping ratio $\gamma_i$ is defined for each of the modes i and matrix $\underset{\sim}{C}_I$ defined as

$$\underset{\sim}{C}_I = (\underset{\sim}{X}_N^{-1})^T \begin{bmatrix} 2\gamma_1 p_1 & & & & \\ & \ddots & & & \\ & & 2\gamma_i p_i & & \\ & & & \ddots & \\ & & & & 2\gamma_n p_n \end{bmatrix} \underset{\sim}{X}_N^{-1} \qquad (4.2\text{-}2)$$

where $p_i$ is the natural circular frequency for mode i, n is the number of modes considered, and $\underset{\sim}{X}_N$ is the modal matrix normalized with respect to the mass matrix.

In the proportional damping approach, matric $\underset{\sim}{C}_I$ is taken to be a linear combination of the mass $\underset{\sim}{M}$ and stiffness $\underset{\sim}{S}$ arrays as

$$\underset{\sim}{C}_I = a\underset{\sim}{M} + b\underset{\sim}{S} \tag{4.2-3}$$

in which a and b are constants of proportionality determined by specifying damping in any two modes. In cases in which the proportional damping formulation was used, constants a and b were determined by specifying damping ratios $\gamma_1$ and $\gamma_2$ for modes 1 and 2.

For internal damping, studies of damping in slender frameworks similar to the prototype structure reveal that a range of 0.5% to 2% is reasonable. In this study, damping ratios of 0.005 and 0.010 were selected to represent the level of internal damping and were used to construct matrix $\underset{\sim}{C}_I$.

Discrete Damping Devices. - - The discrete damper matrix $\underset{\sim}{C}_D$ for the entire tower structure was developed from individual member damping matrices for add-on damper elements connecting any two tower nodes. The damper device was assumed to be a part of an additional diagonal bracing element with the capability of two way action. For example, the arrangement shown in Figure 4.2-1 was assumed to be able to generate a damping force proportional to the relative velocity between nodes i and j during either tension or compression of the element. The device was assumed to add no mass or stiffness to the structure.

The damping element itself may, for example, be a conventional automobile shock absorber with suitable valving to operate in the appropriate frequency range. Frequency and amplitude-dependent characteristics of the device were

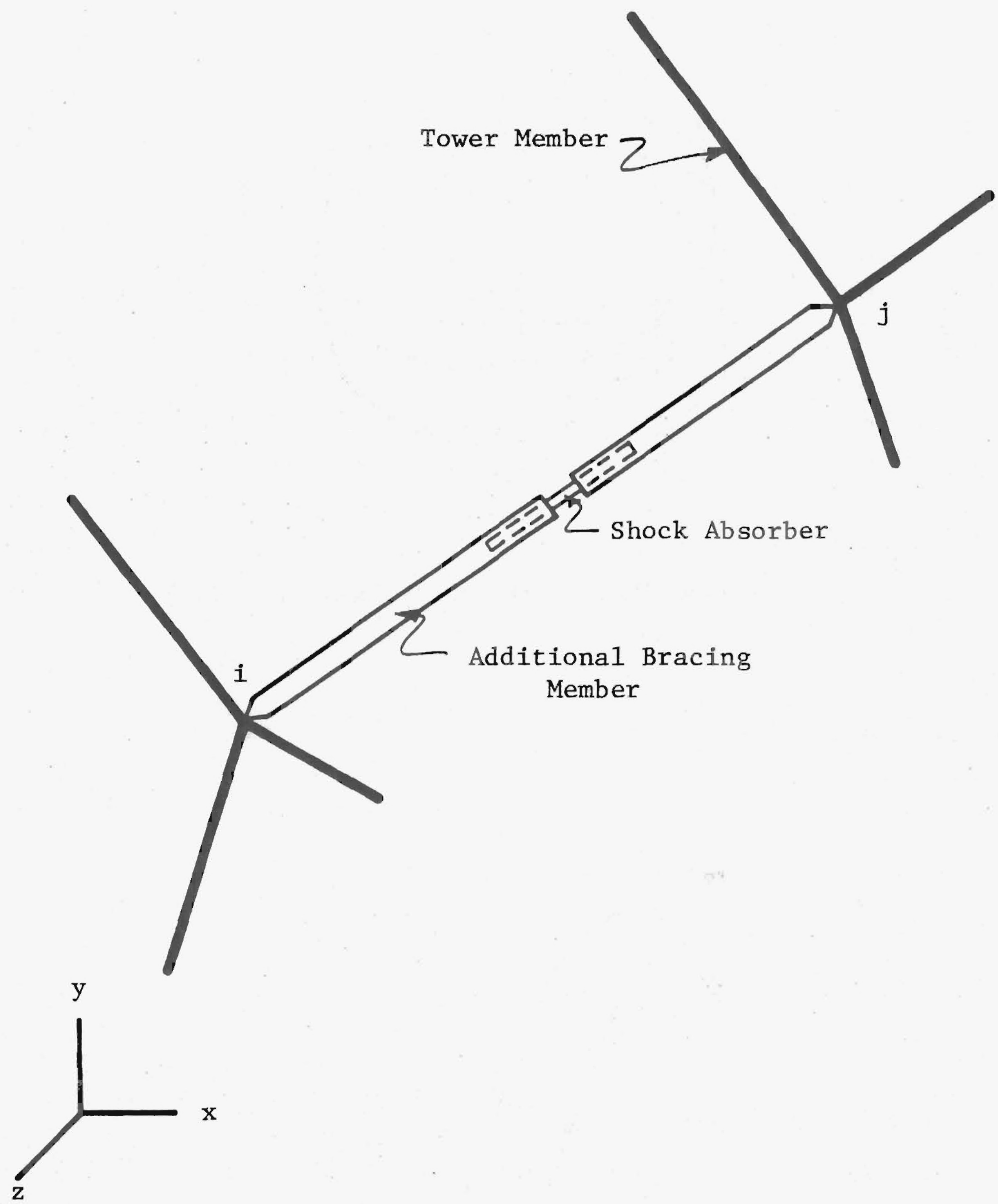



Figure 4.2-1 - Model of Energy Absorbing Device

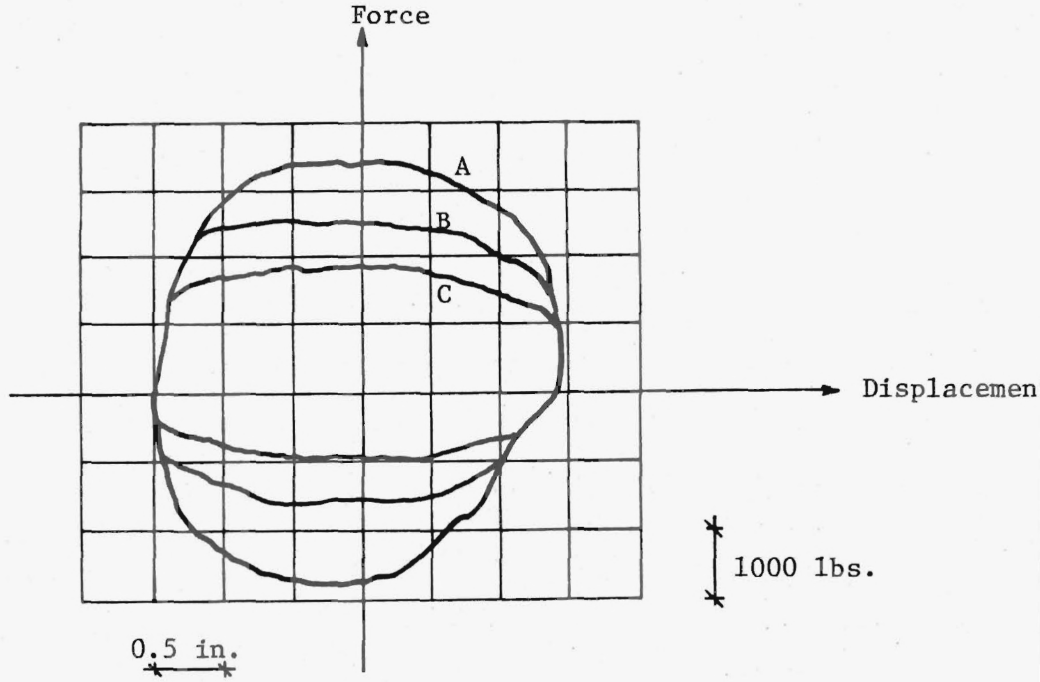
ignored for simplicity and its damping property characterized by an equivalent viscous damping constant $C_{eq}$.

Hysteresis loops for a commercially available shock absorber, tested in three frequency ranges and at a maximum amplitude of 1.5 in. (3.8 cm), are presented in Figure 4.2-2(a). A typical loop is formed by the stress-strain curve for increasing and decreasing levels of stress and strain, and Figure 4.2-2(b) shows a complete reversal of stress and strain corresponding to one cycle of vibration. The internal damping mechanism dissipates energy approximately in proportion to the square of the strain amplitude and the shape of the hysteresis loop is relatively independent of the amplitude and strain rate [65].

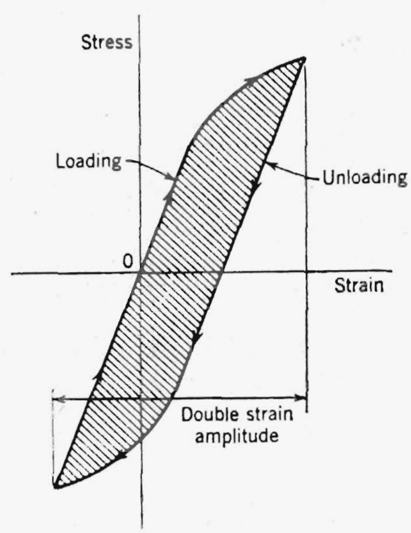
By equating the energy dissipated per cycle by the shock absorber to the work done by a sinusoidal disturbing force per cycle during steady-state response, a range of $C_{eq}$ values were determined. The values obtained for the loops of Figure 4.2-2(a) are: 0.464 k-sec/in (0.811 kN-sec/cm) for loop A, 0.215 k-sec/in (0.376 kN-sec/cm) for loop B, and 0.164 k-sec/in (0.286 kN-sec/cm) for loop C. Based on these calculated values, a range of 0.1 to 0.5 k-sec/in (0.174 to 0.871 kN-sec/cm) was selected for use in the parameter studies.

The equivalent viscous damping constant $C_{eq}$ was then used to assemble a member damping matrix $\underset{\sim}{C}_M$ with respect to the local member axes coordinate system. The form of $\underset{\sim}{C}_M$ is the same as the member stiffness matrix for a space truss element [22] and is expressed as

$$\underset{\sim}{C}_M = C_{eq} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (4.2\text{-}4)$$




(a) Hysteresis Loops for Shock Absorber




(b) Typical Hysteresis Loop (taken from Reference 65)

Figure 4.2-2 - Hysteresis Loops

A rotation of axes transformation of the form

$$C_{MD} = R_T^T \, C_M \, R_T \tag{4.2-5}$$

was used to develop $C_{MD}$, the member damping matrix with respect to structure axes. Matrix $R_T$ is the rotation transformation matrix for a space truss element [22]. Finally, the discrete damper matrix $C_D$ was developed as the assemblage of matrices $C_{MD}$ for all damper elements in the structure.

In general, the damper element can be placed between any two tower nodes which contain master degrees of freedom. The direct assemblage model of the large tower, described in Chapter 3, was composed of 21 substructures with 63 master degrees of freedom, 3 per level. To further simplify the add-on damper model described above, damping devices were assumed to be connected in the x-direction only between the 21 levels in the tower model, rather than between specific joints in the tower.

A variety of internal and add-on damping cases were then formulated, and the influence of damping on the x-direction seismic response of the large tower determined.

Damping Cases. - - The fourteen different damping cases considered in the parametric study of tower response to earthquake loading are summarized in Table 4.2-1. A different damping matrix $C_T$ was assembled for each case and the effect of varying the number, size, and distribution of discrete dampers was determined.

Case 0 is the reference case with no damping present in the tower model. Cases 1 to 4 consist of internal damping only, using both simple modal and proportional damping, and were used for comparison to cases 5 to 13 containing both internal and add-on damping. In cases 5 to 8, simple modal internal damping was combined with either 10 or 20 discrete dampers of uniform size connected

TABLE 4.2-1 - Damping Cases

| Case number | Internal damping formulation | Damping ratio and modes | No. of dampers | Location | Damper size $C_{eq}$ |
|---|---|---|---|---|---|
| 0 | Undamped | 0.0 for all modes | 0 | - | - |
| 1 | Simple modal | 0.005 for all modes | 0 | - | - |
| 2 | Simple modal | 0.01 for all modes | 0 | - | - |
| 3 | Proportional (Rayleigh) | 0.005 modes 1 and 2 | 0 | - | - |
| 4 | Proportional | 0.01 modes 1 and 2 | 0 | - | - |
| 5 | Simple modal | 0.01 for all modes | 20 | Every level* | 0.10 k-sec/in.** |
| 6 | Simple modal | 0.01 for all modes | 10 | Every other level* | 0.10 k-sec/in. |
| 7 | Simple modal | 0.005 for all modes | 20 | Every level* | 0.10 k-sec/in. |
| 8 | Simple modal | 0.005 for all modes | 10 | Every other level* | 0.10 k-sec/in. |
| 9 | Simple modal | 0.01 for all modes | 20 | Every level* | 0.25 k-sec/in. |
| 10 | Simple modal | 0.01 for all modes | 20 | Every level* | 0.50 k-sec/in. |
| 11 | Simple modal | 0.01 for all modes | 20 | Every level (linear distrib.) | 0.029 k-sec/in at top<br>0.179 k-sec/in at base |
| 12 | Simple modal | 0.01 for all modes | 20 | Every level (linear distrib.) | 0.179 k-sec/in at top<br>0.029 k-sec/in at base |
| 13 | Simple modal | 0.01 for all modes | 20 | Every level (unif. distrib. at top 3 sections, linear from there to base) | 0.333 k-sec/in first 3<br>0.118 k-sec/in fourth<br>0.000 at base |

* Uniform distribution

** 1 k-sec/in = 1.751 kN-sec/cm

between every level, or every other level, in the model of the large tower. In cases 9 and 10, the size of all dampers was increased while internal damping was held constant. Finally, in cases 11, 12, and 13, damper size was varied over the height of the tower, as shown in Figure 4.2-3. To permit a direct comparison to be made between cases 5, 11, 12, and 13, the total damping force was held constant for these cases. The total force was computed as the sum of the products of damper size and corresponding velocity change for an assumed uniform differential velocity distribution over the height of the tower.

## 4.3 Dynamic Response Analysis

The time-history dynamic response of the protytype structure was computed for several moderate seismic loadings to evaluate the effectiveness of discrete damping devices in reducing structure response. In particular, the size, number and distribution of dampers required to attenuate structure dynamic response were sought in these studies.

In this section, the computational procedures used to determine structure response to moderate earthquake ground motion are described first. Then, structure displacement-time histories for the variety of different damping cases described in Table 4.2-1 above are presented. Finally, the effectiveness of add-on damping was evaluated using several different response attenuation measures.

Computational Model. - - The dynamic response of the reduced model was obtained by a step-by-step integration procedure. In this approach, the response was evaluated for a series of short time increments, $\Delta t$, taken of equal length for computational convenience. The condition of dynamic equilibrium was established at the beginning and at the end of each interval, and the motion of the system during the time increment was evaluated approximately on the

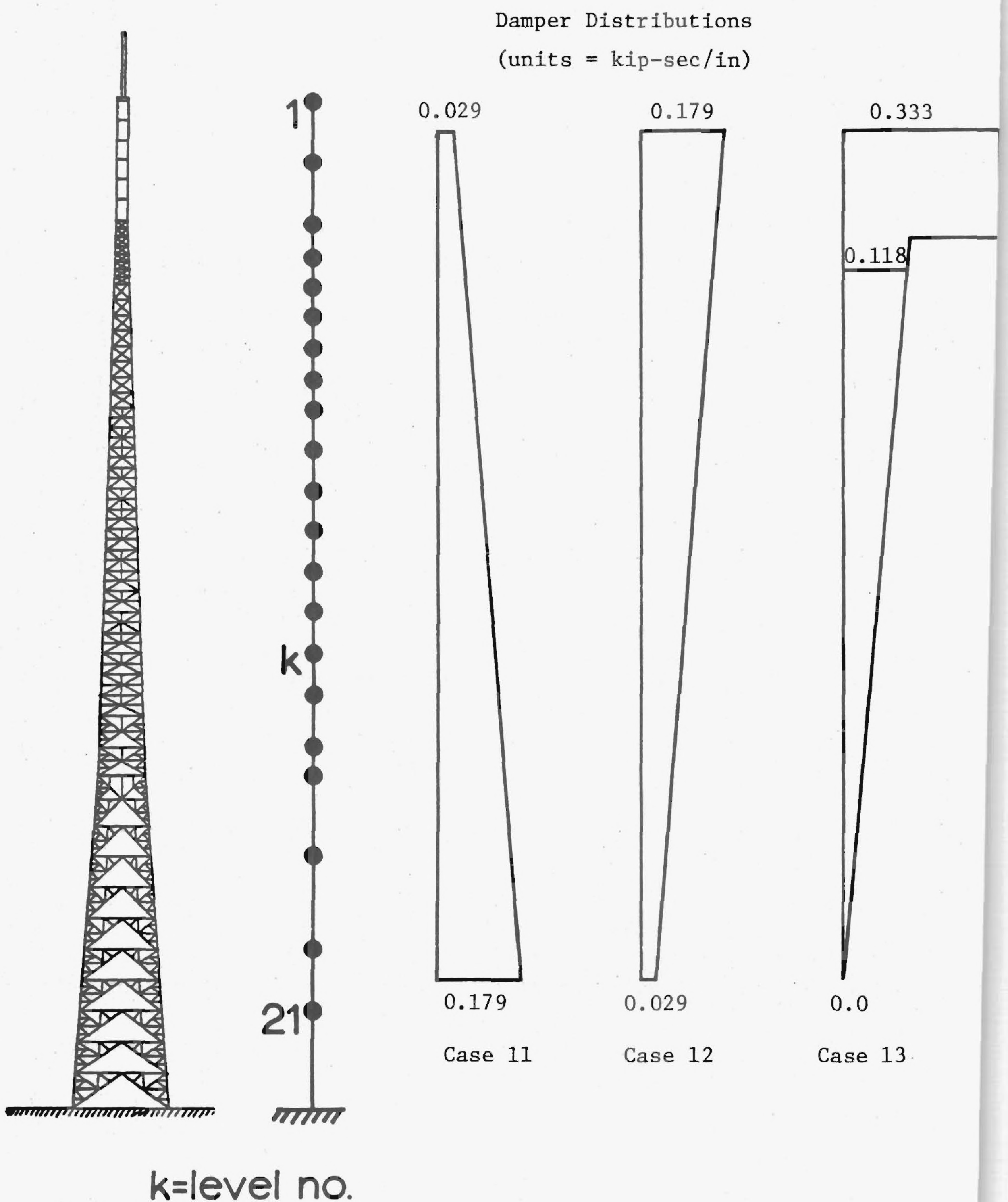



Figure 4.2-3 - Damper Distributions for Cases 11, 12, and 13

basis of an assumed linear distribution of displacement and velocity. This procedure typically ignores the lack of equilibrium which may develop during the interval. The complete response was obtained by using the velocity and displacement computed at the end of one interval as the initial conditions for the next interval; thus the process was continued step-by-step from the initiation of loading to the maximum time of interest. A FORTRAN computer program was written to do the step-by-step computations and is listed in Appendix C.

The stiffness and mass matrices for the direct assemblage model were used in the dynamic response analysis. Loadings were described using piecewise-constant interpolation, and the value of the loading at the center of the interval under consideration was used to approximate the loading during the entire interval [65]. It was assumed that wind, seismic, and harmonic disturbing forces could be adequately described in this manner.

<u>Direct Linear Extrapolation with the Trapezoidal Rule</u>. - - A number of numerical integration procedures were available for the solution of the equations of motion [1]

$$\underset{\sim}{M}\,\ddot{\underset{\sim}{X}} + \underset{\sim}{C}\,\dot{\underset{\sim}{X}} + \underset{\sim}{S}\,\underset{\sim}{X} = \underset{\sim}{A} \qquad (4.3\text{-}1)$$

where $\underset{\sim}{M}$, $\underset{\sim}{C}$ and $\underset{\sim}{S}$ are the mass, damping and stiffness matrices respectively for the assembled structure. Vectors $\underset{\sim}{X}$, $\dot{\underset{\sim}{X}}$, $\ddot{\underset{\sim}{X}}$, and $\underset{\sim}{A}$ represent displacements, velocities, accelerations and actions at the degrees of freedom, respectively.

The direct linear extrapolation technique with the trapezoidal rule was used to solve for total displacements of the structure at each time step. In applying this technique, uniform time steps were used and total response evaluated at the end of each step. Velocities were approximated at the end of the step from the trapezoidal rule as

$$\dot{\underset{\sim}{X}}_{i} = \dot{\underset{\sim}{X}}_{i-1} + \left( \ddot{\underset{\sim}{X}}_{i-1} + \ddot{\underset{\sim}{X}}_{i} \right) \frac{\Delta t}{2} \tag{4.3-2}$$

and displacements were assumed to be

$$\underset{\sim}{X}_{i} = \underset{\sim}{X}_{i-1} + \left( \dot{\underset{\sim}{X}}_{i-1} + \dot{\underset{\sim}{X}}_{i} \right) \frac{\Delta t}{2} \tag{4.3-3}$$

where subscripts indicate current response time point i and prior time point i-1. Substituting Equation (4.3-2) into Equation (4.3-3) results in

$$\underset{\sim}{X}_{i} = \underset{\sim}{X}_{i-1} + \dot{\underset{\sim}{X}}_{i-1} \Delta t + \left( \ddot{\underset{\sim}{X}}_{i-1} + \ddot{\underset{\sim}{X}}_{i} \right) \frac{(\Delta t)^2}{4} \tag{4.3-4}$$

The linear damped equations of motion (see Equation (4.3-1)) for the i-th time point are

$$\underset{\sim}{M} \ddot{\underset{\sim}{X}}_{i} + \underset{\sim}{C} \dot{\underset{\sim}{X}}_{i} + \underset{\sim}{S} \underset{\sim}{X}_{i} = \underset{\sim}{A}_{i} \tag{4.3-5}$$

Solving for $\dot{X}_i$ in Equation (4.3-3) leads to

$$\dot{\underset{\sim}{X}}_{i} = \left( \underset{\sim}{X}_{i} - \underset{\sim}{X}_{i-1} - \dot{\underset{\sim}{X}}_{i-1} \frac{\Delta t}{2} \right) \frac{2}{\Delta t} \tag{4.3-6}$$

and solving for $\ddot{X}_i$ in Equation (4.3-4) gives

$$\ddot{\underset{\sim}{X}}_{i} = \left( \underset{\sim}{X}_{i} - \underset{\sim}{X}_{i-1} - \dot{\underset{\sim}{X}}_{i-1} \Delta t - \ddot{X}_{i-1} \frac{(\Delta t)^2}{4} \right) \frac{4}{(\Delta t)^2} \tag{4.3-7}$$

Substituting Equations (4.3-6) and (4.3-7) into Equation (4.3-5) and collecting terms results in

$$\underset{\sim}{S}^{*} \underset{\sim}{X}_{i} = \underset{\sim}{A}_{i}^{*} \tag{4.3-8}$$

where

$$\underset{\sim}{S}^{*} = \underset{\sim}{S} + \underset{\sim}{M} \frac{4}{(\Delta t)^2} + \underset{\sim}{C} \frac{2}{\Delta t} \tag{4.3-9}$$

and

$$\underset{\sim}{A}_i^* = \underset{\sim}{A}_i + \underset{\sim}{M} \left( \underset{\sim}{X}_{i-1} + \dot{\underset{\sim}{X}}_{i-1} \Delta t + \ddot{\underset{\sim}{X}}_{i-1} \frac{(\Delta t)^2}{4} \right) \frac{4}{(\Delta t)^2} + \underset{\sim}{C} \left( \underset{\sim}{X}_{i-1} + \dot{\underset{\sim}{X}}_{i-1} \frac{\Delta t}{2} \right) \frac{2}{\Delta t} \tag{4.3-10}$$

By letting

$$\underset{\sim}{Q}_{i-1} = \left( \underset{\sim}{X}_{i-1} + \dot{\underset{\sim}{X}}_{i-1} \Delta t + \ddot{\underset{\sim}{X}}_{i-1} \frac{(\Delta t)^2}{4} \right) \frac{4}{(\Delta t)^2} \tag{4.3-11}$$

and

$$\underset{\sim}{P}_{i-1} = \left( \underset{\sim}{X}_{i-1} + \dot{\underset{\sim}{X}}_{i-1} \frac{\Delta t}{2} \right) \frac{2}{\Delta t} \tag{4.3-12}$$

Equation (4.3-10) was reduced to

$$\underset{\sim}{A}_i^* = \underset{\sim}{A}_i + \underset{\sim}{C}\, \underset{\sim}{P}_{i-1} + \underset{\sim}{M}\, \underset{\sim}{Q}_{i-1} \tag{4.3-13}$$

Equation (4.3-6) became

$$\dot{\underset{\sim}{X}}_i = \underset{\sim}{X}_i \frac{2}{\Delta t} - \underset{\sim}{P}_{i-1} \tag{4.3-14}$$

and Equation (4.3-7) simplified to

$$\ddot{\underset{\sim}{X}}_i = \underset{\sim}{X}_i \frac{4}{(\Delta t)^2} - \underset{\sim}{Q}_{i-1} \tag{4.3-15}$$

The solution algorithm can be summarized as follows:

(a) Initialize $\underset{\sim}{X}_i$ and $\dot{\underset{\sim}{X}}_i$ to produce displacements $\underset{\sim}{X}_o$, and velocities $\dot{\underset{\sim}{X}}_o$ at time zero (i = 0); compute $\ddot{\underset{\sim}{X}}_o$ ( $\ddot{\underset{\sim}{X}}_i$ at i = 0 ) from the equation of motion

$$\ddot{\underset{\sim}{X}}_o = \underset{\sim}{M}^{-1} \left( \underset{\sim}{A}_o - \underset{\sim}{C}\, \dot{\underset{\sim}{X}}_o - \underset{\sim}{S}\, \underset{\sim}{X}_o \right) \tag{4.3-16}$$

(b) Compute $\underset{\sim}{S}^*$ and decompose. $\underset{\sim}{S}^* = \underset{\sim}{U}'\, \underset{\sim}{U}$ using the Cholesky method, where $\underset{\sim}{U}$ is an upper triangular matrix.

(c) Perform the following sequence of calculations for time step i ( $\geq 1$ )

(1) Compute $\underset{\sim}{P}_{i-1}$

(2) Compute $\underset{\sim}{Q}_{i-1}$

(3) Compute $\underset{\sim}{A}_i^*$

(4) Solve for $\underset{\sim}{X}_i$ in two steps

$$\underset{\sim}{U}' \underset{\sim}{X}_i^* = \underset{\sim}{A}_i^* \qquad \text{(forward solution)}$$

$$\underset{\sim}{U} \underset{\sim}{X}_i = \underset{\sim}{X}_i^* \qquad \text{(backward solution)}$$

(5) Compute $\dot{\underset{\sim}{X}}_i$

(6) Compute $\ddot{\underset{\sim}{X}}_i$

(7) Go to (1) and continue

Repetitive use of an approximation formula can cause error accumulation that may artifically magnify or attenuate the response of a structure. Care should be taken to ensure that roundoff error is small. For some step-by-step integration procedures, if the time step is chosen too large, the solution for response of the structure becomes unstable. Direct linear extrapolation with the trapezoidal rule is unconditionally stable for all time steps. However, as a general rule, the time increment should be taken to be less than one-tenth of the period of the highest significant mode of the structure. In addition, the nature of the forcing function must be considered; a $\Delta t$ which is less than or equal to the time interval at which the forcing function has been interpolated should be chosen.

The load-time history matrix $\underset{\sim}{A}$ for the degrees of freedom was evaluated using linear interpolation of independent time functions in the structure x, y, and z directions. The values in $\underset{\sim}{A}$ were taken as the products of load factors and the time function values at the center of each time interval. In the case of ground acceleration input, the load factors consisted of the negative values of mass-inertia terms associated with the dynamic degrees of freedom.

Selected Loading. - - The response of the prototype structure was determined using several moderate seismic loadings. The NOOE component of the 1971 San Fernando earthquake, recorded at Ft. Tejon, California, on February 9, at 6:00 a.m. P.S.T., was selected as the principal excitation to be used in the evaluation of damper effectiveness. A plot of this record is contained in Figure 4.3-1. The Ft. Tejon record was selected because of its short duration (10.4 seconds) and its moderate size, resulting in reduced computer run time and linear elastic structure response.

Parameter Study Results. - - The fourteen different damping cases summarized in Table 4.2-1 were used to evaluate the effect of add-on dampers on the seismic response of the prototype structure. The structure response at degrees of freedom 1, 28 and 52 (see Figure 3.2-4) to the selected earthquake record for damping case 2 (Table 4.2-1) is shown in Figure 4.3-2. Comparison of the response at the three different degrees of freedom showed that the displacement response at degree of freedom 52 was relatively small compared to the displacement at degrees of freedom 1 and 28. This indicated that the structure response was concentrated in the flexible part of the tower above the first bend line.

The responses of the prototype structure at degrees of freedom 1 and 28 to the selected seismic loading for the damping cases listed in Table 4.2-1 are shown in Figures 4.3-3 through 4.3-14. In general, these figures show that the effect of internal damping was small compared to the effect of discrete dampers of the size used in this study and that the effect of add-on dampers was more noticeable during free vibration response than during forced-excitation response. The responses differ very little for the simple modal

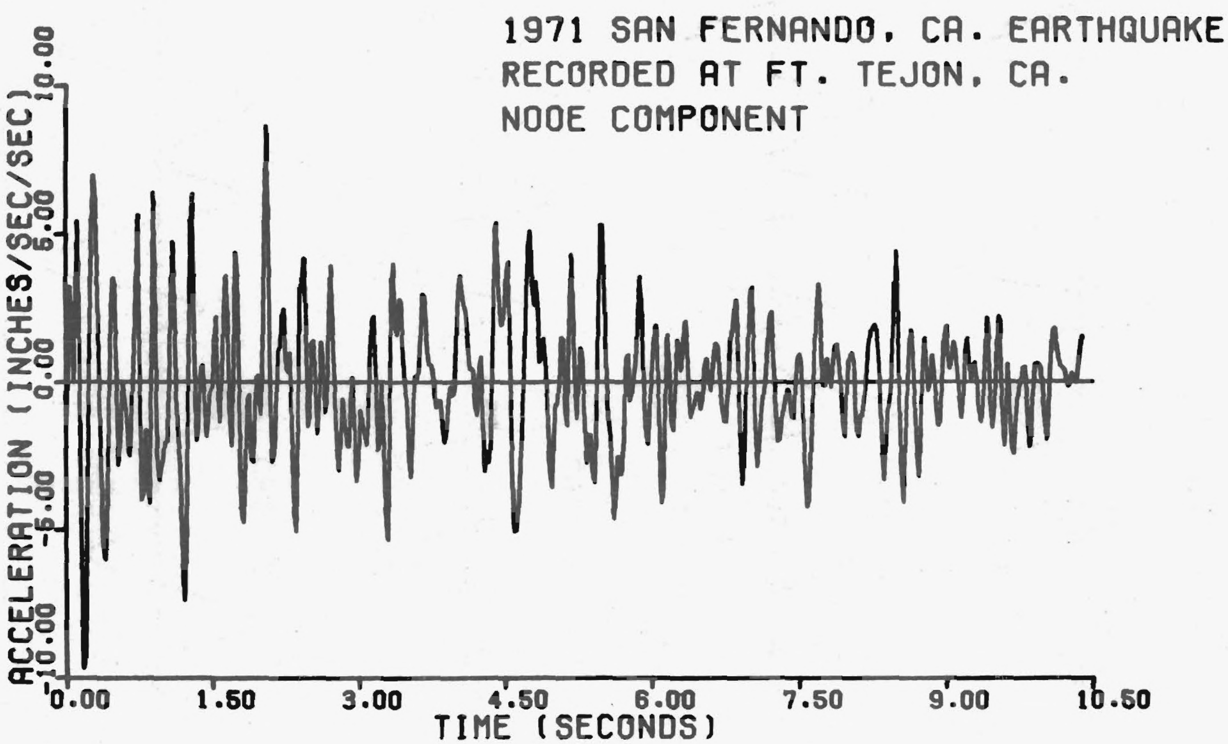



Figure 4.3-1 - NOOE Component of the 1971 San Fernando Earthquake Recorded at Ft. Tejon, California, on February 9 at 6:00 a.m. PST.

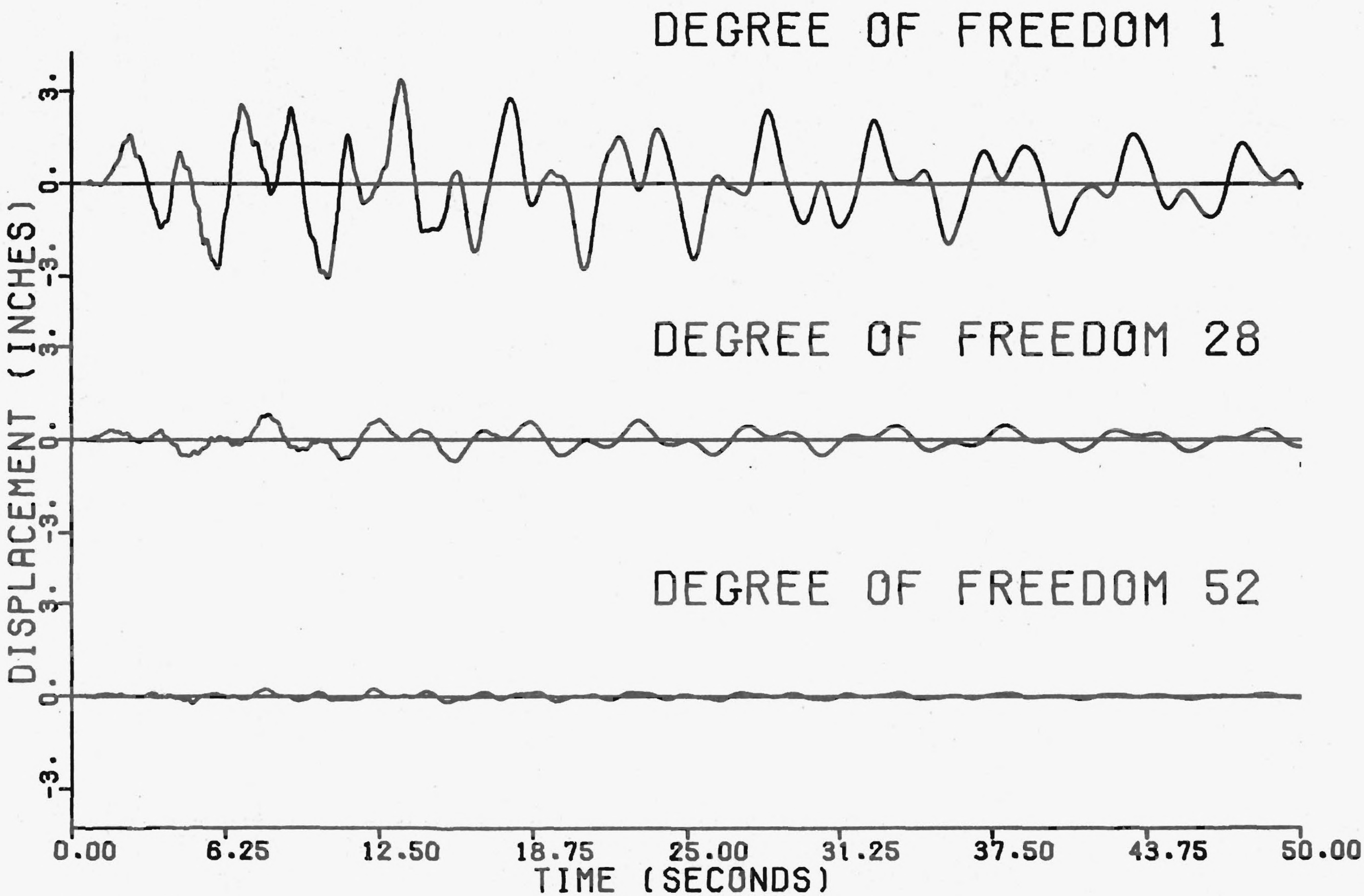



Figure 4.3-2 - Structure Response to the Ft. Tejon Record for Case 2 Damping

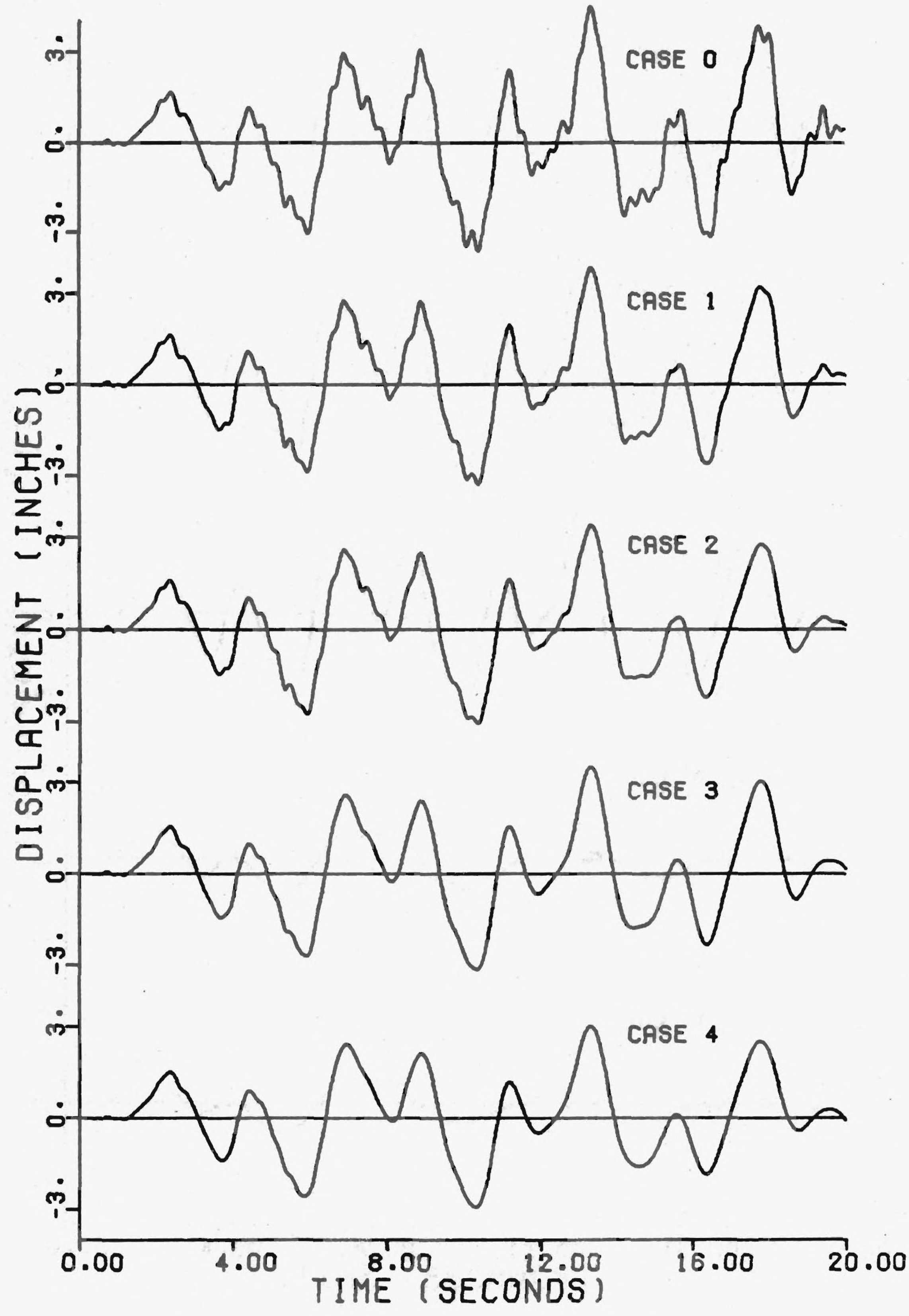



Figure 4.3-3 - Structure Response at Degree of Freedom 1 to the 10.4 second Ft. Tejon Record, Damping Cases 0 to 4.

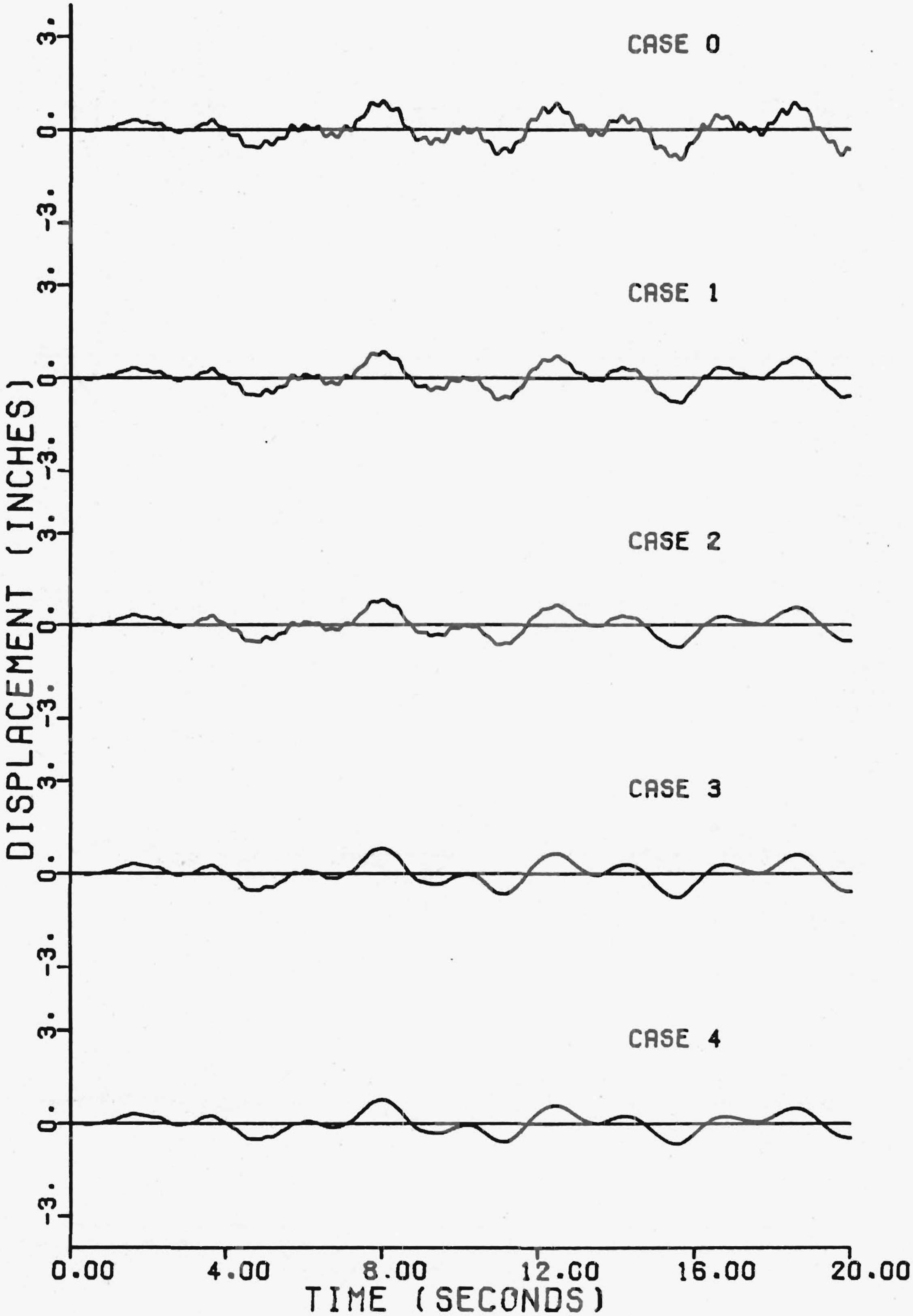



Figure 4.3-4 - Structure Response at Degree of Freedom 28 to the 10.4 second Ft. Tejon Record, Damping Cases 0 to 4.

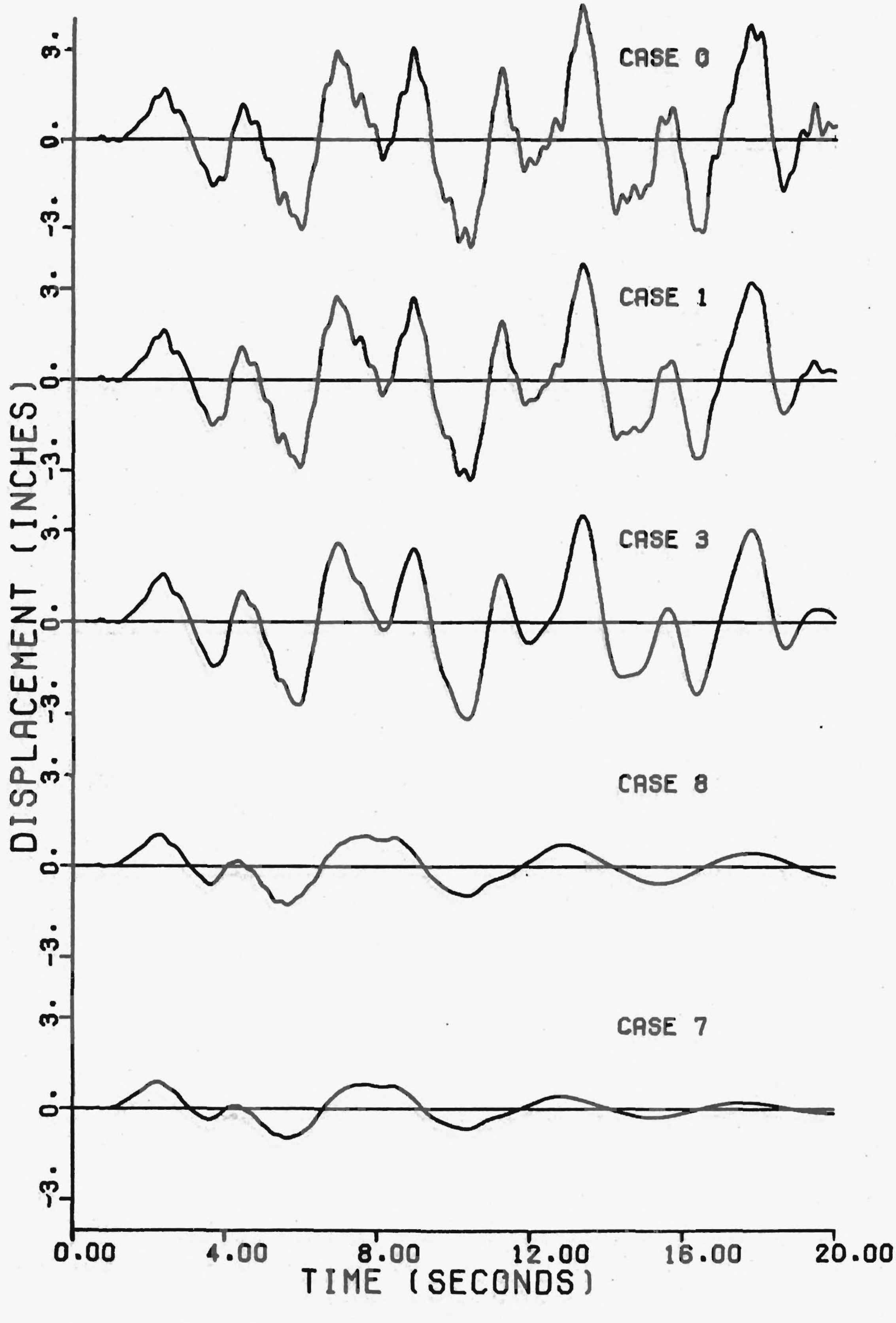



Figure 4.3-5 - Structure Response at Degree of Freedom 1 to the 10.4 second Ft. Tejon Record, Damping Cases 0, 1, 3, 7 and 8.

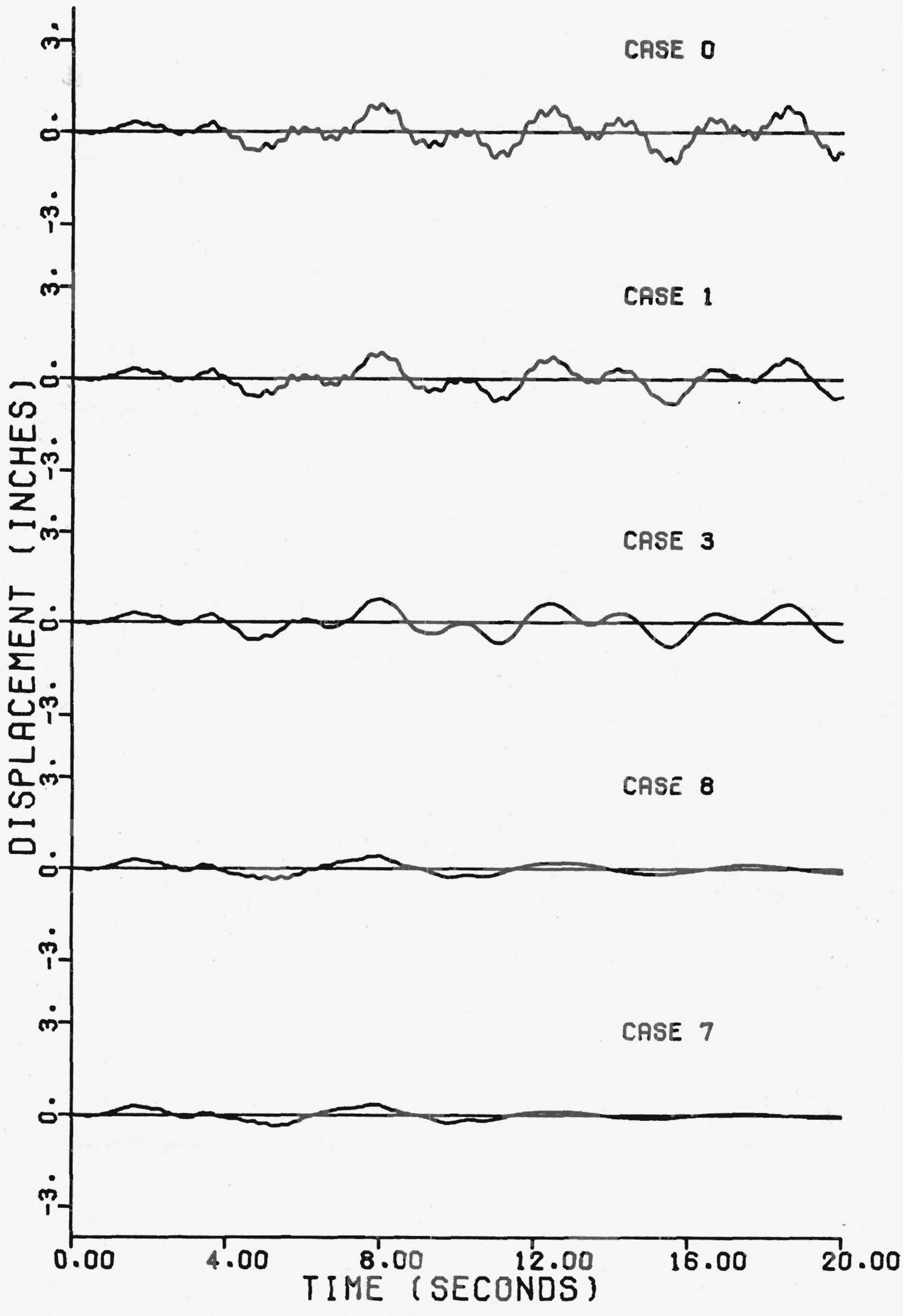



Figure 4.3-6 - Structure Response at Degree of Freedom 28 to the 10.4 second Ft. Tejon Record, Damping Cases 0, 1, 3, 7 and 8.

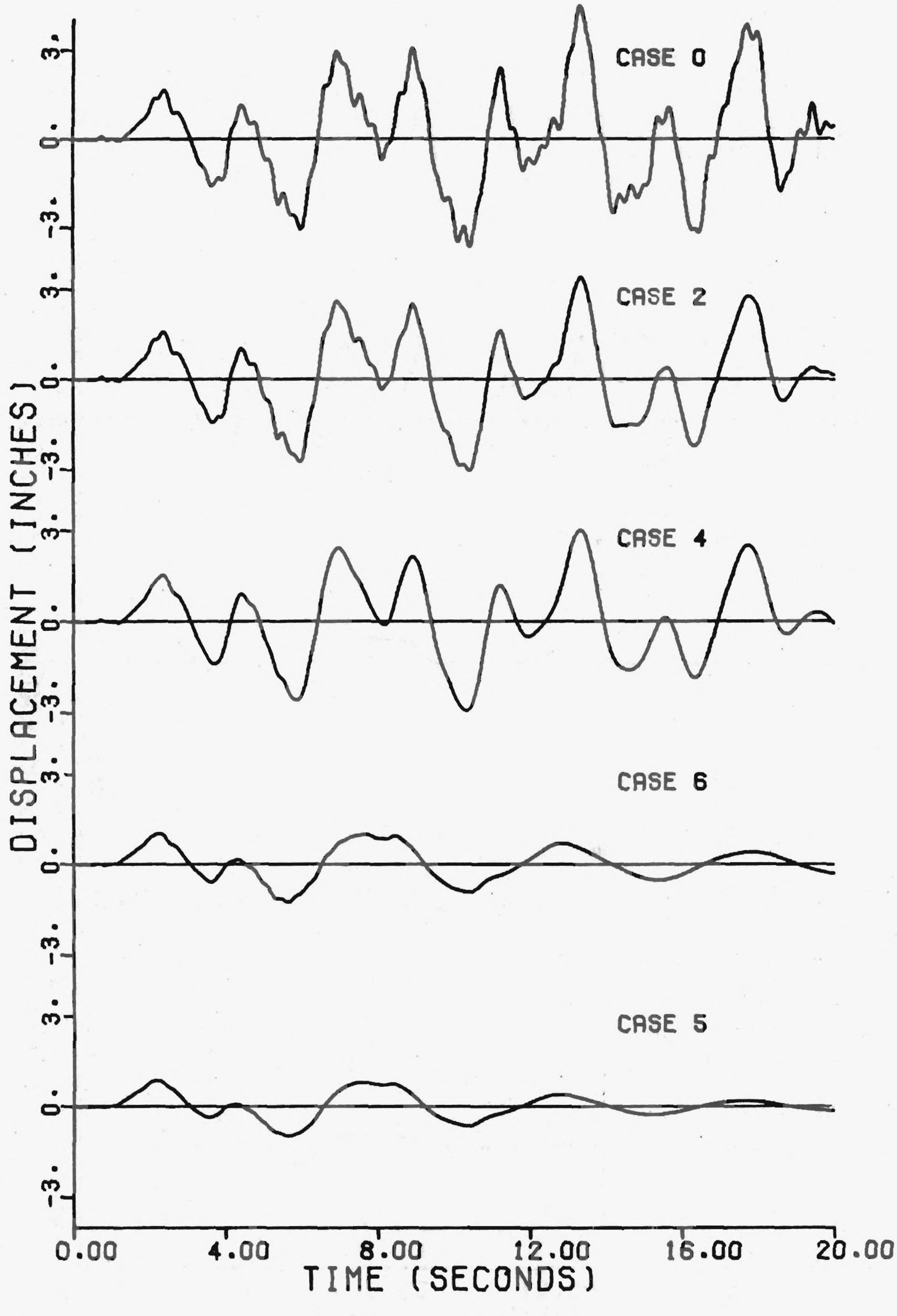



Figure 4.3-7 - Structure Response at Degree of Freedom 1 to the 10.4 second Ft. Tejon Record, Damping Cases 0, 2, 4, 5, and 6.

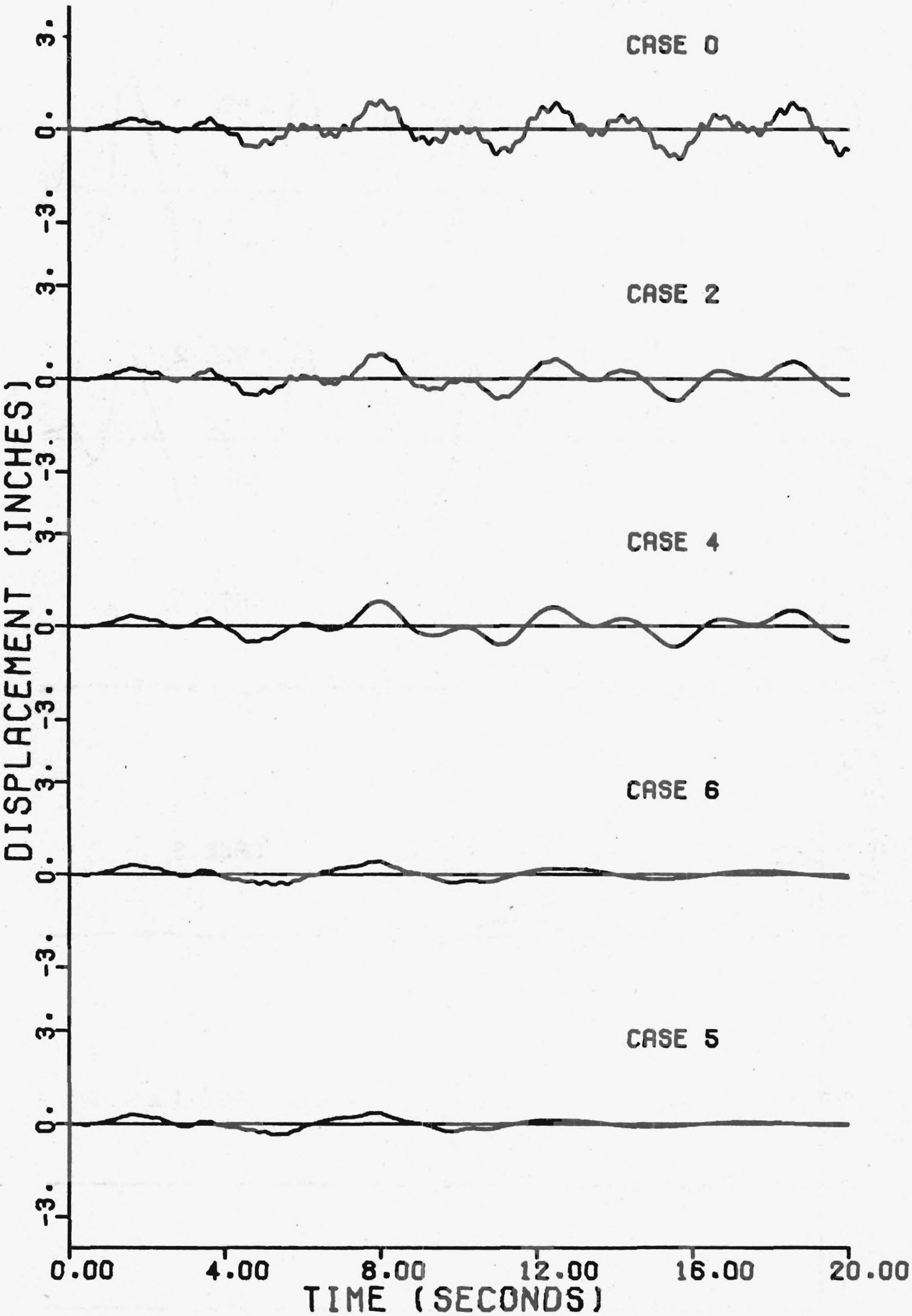



Figure 4.3-8 - Structure Response at Degree of Freedom 28 to the 10.4 second Ft. Tejon Record, Damping Cases 0, 2, 4, 5 and 6.

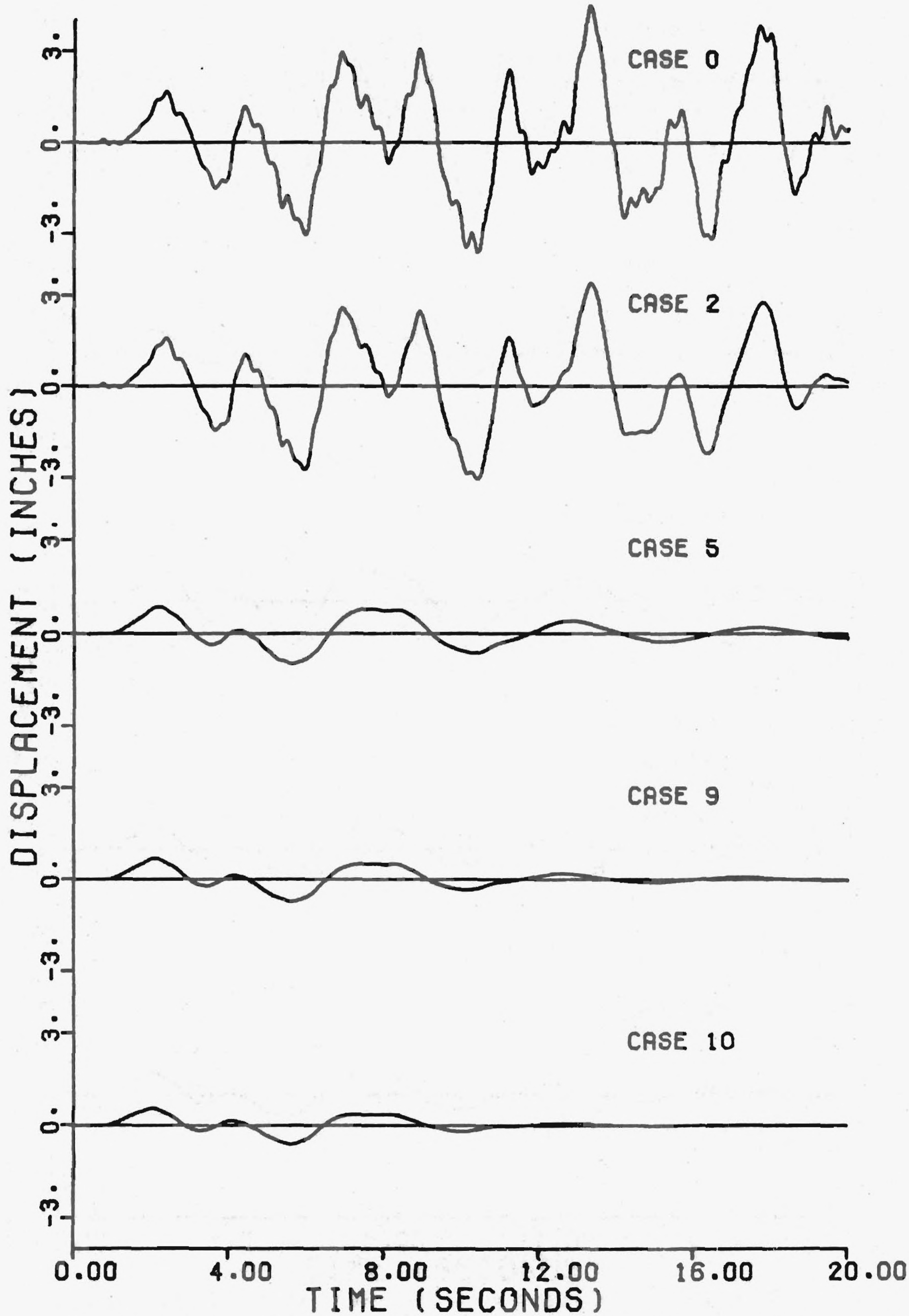



Figure 4.3-9 - Structure Response at Degree of Freedom 1 to the 10.4 second Ft. Tejon Record, Damping Cases 0, 2, 5, 9, and 10.

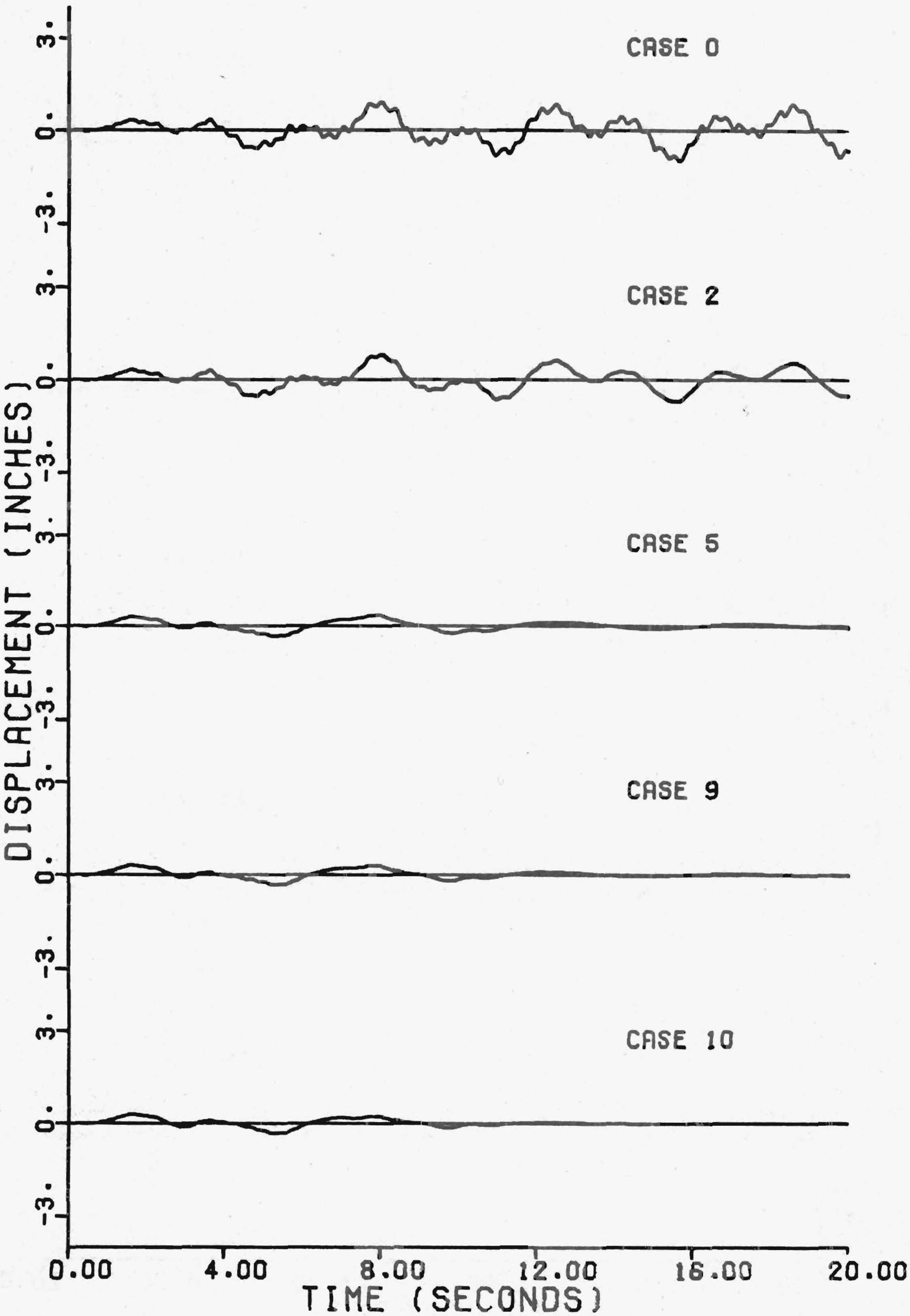



Figure 4.3-10 - Structure Response at Degree of Freedom 28 to the 10.4 second Ft. Tejon Record, Damping Cases 0, 2, 5, 9 and 10.

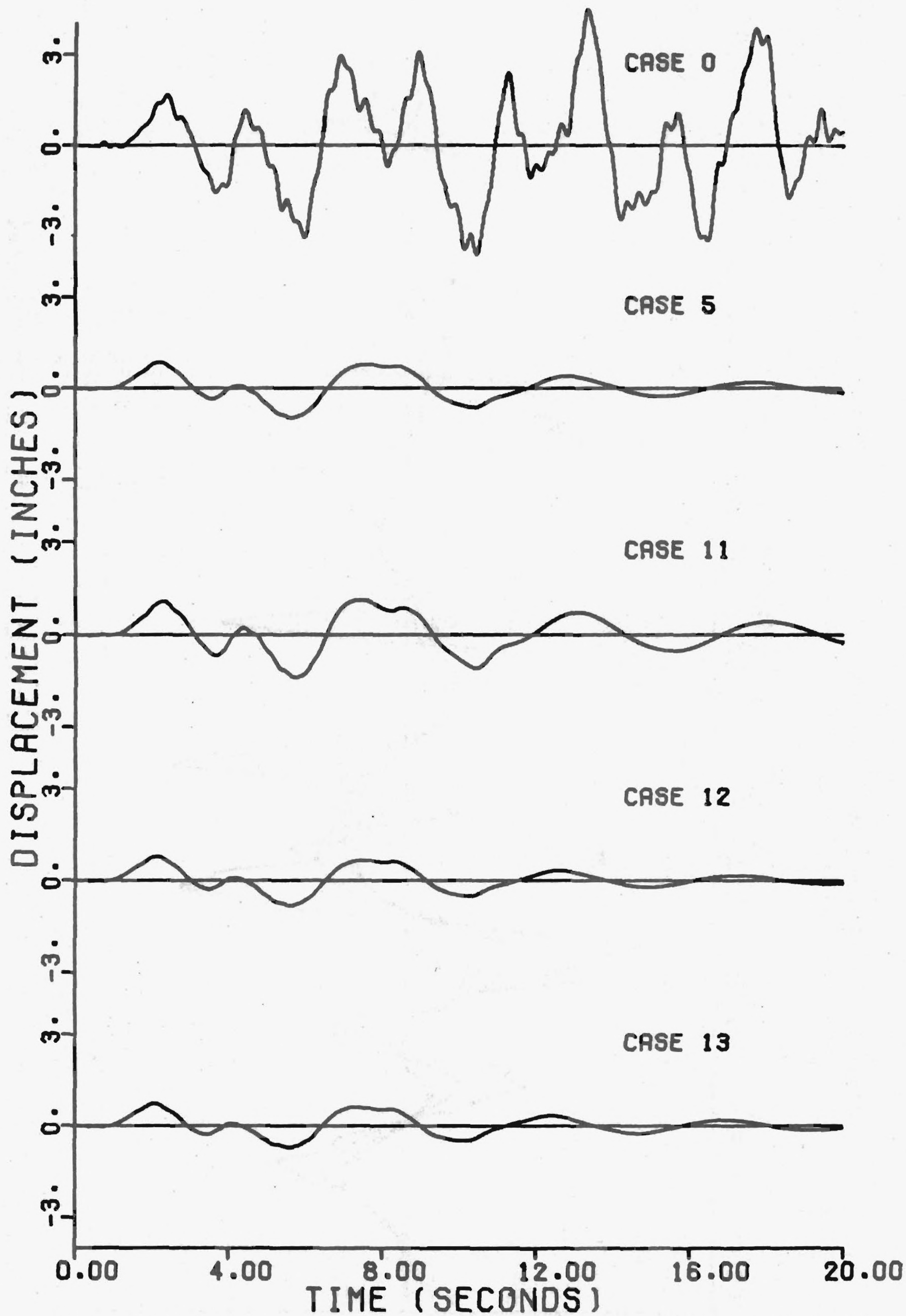



Figure 4.3-11 - Structure Response at Degree of Freedom 1 to the 10.4 second Ft. Tejon Record, Damping Cases 0, 5, 11, 12, and 13.

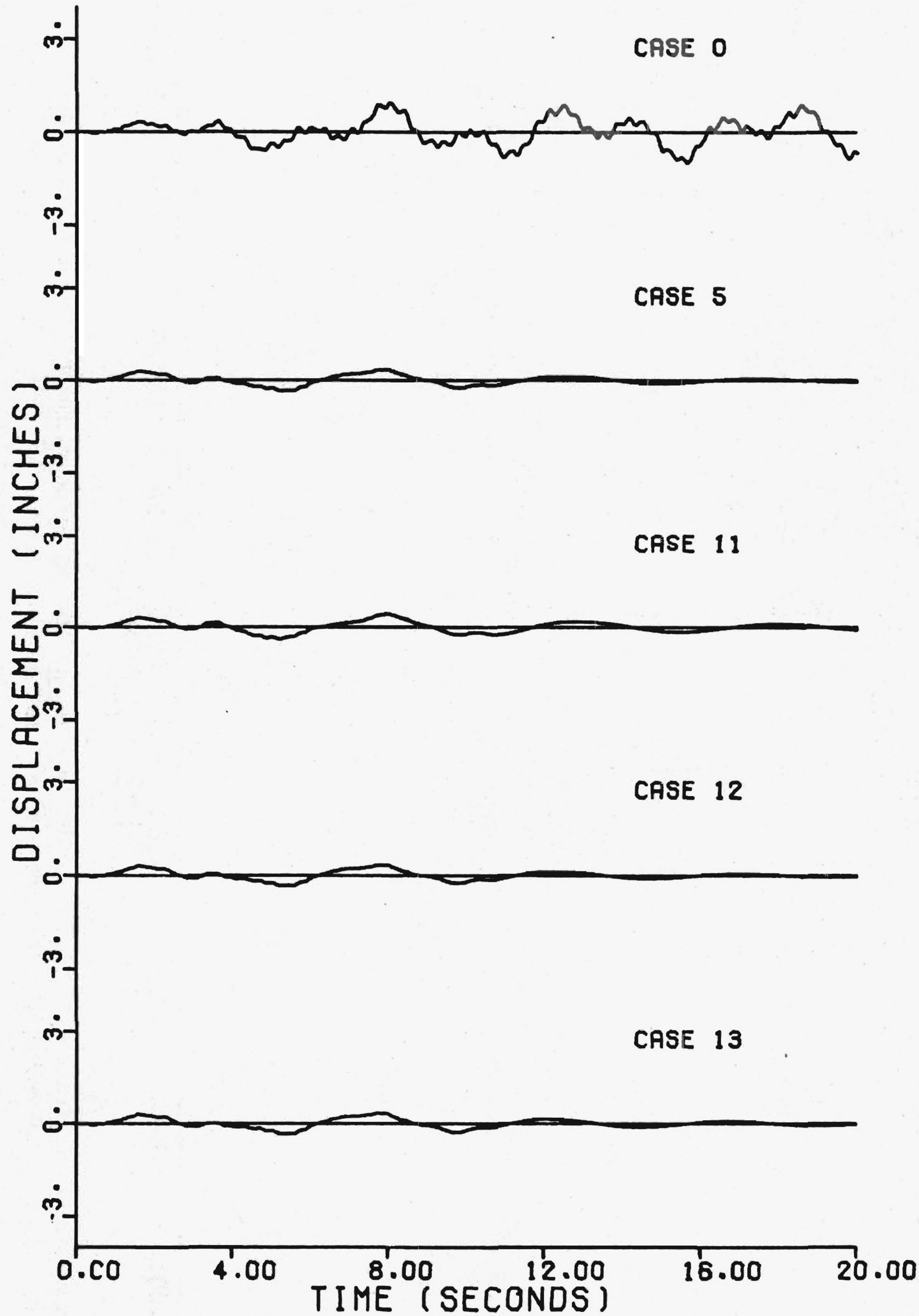



Figure 4.3-12 - Structure Response at Degree of Freedom 28 to the 10.4 second Ft. Tejon Record, Damping Cases 0, 5, 11, 12 and 13.

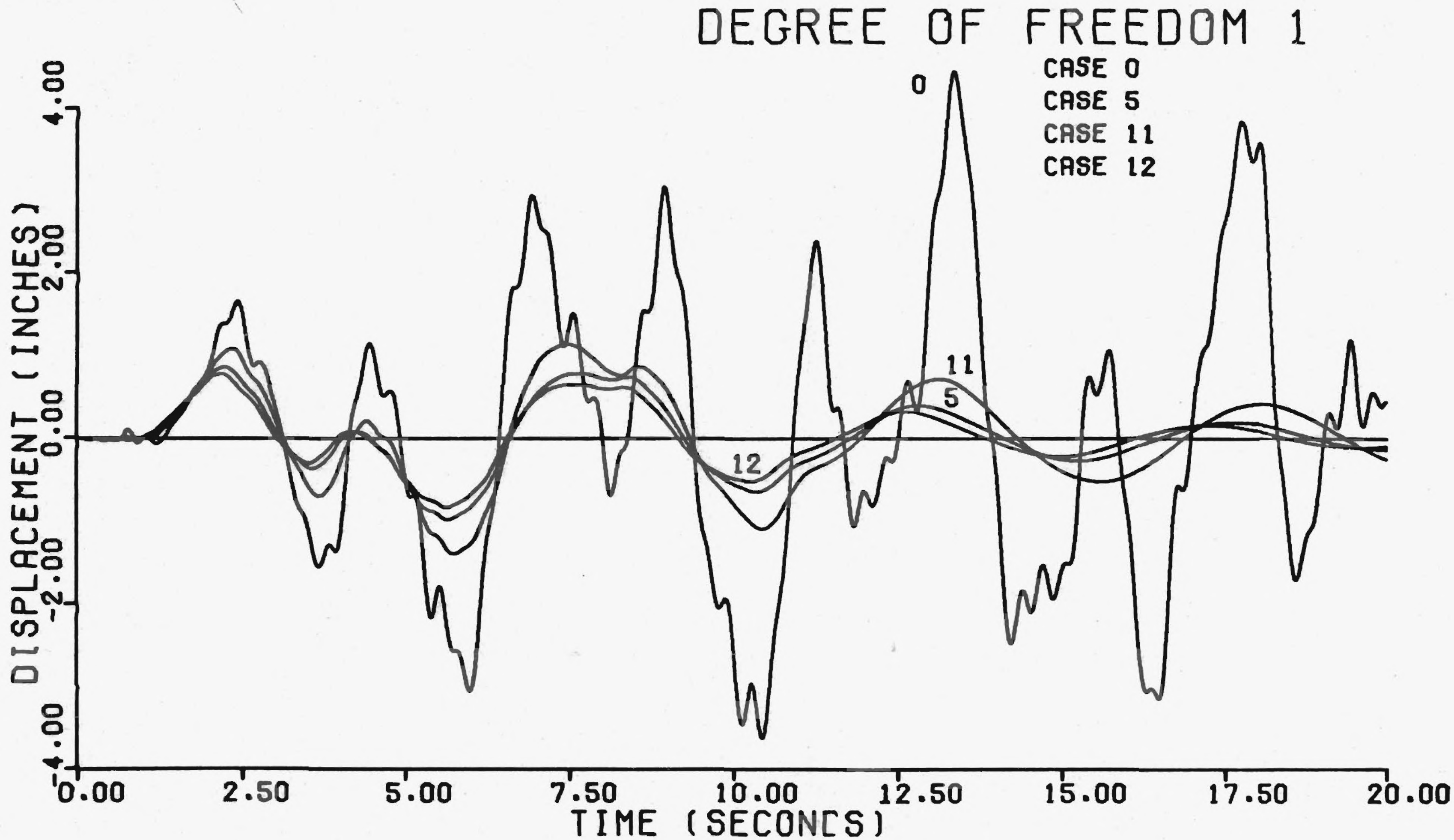



Figure 4.3-13 - Structure Response to the 10.4 second Ft. Tejon Record for Different Cases of Damping (summary of data presented in Figure 4.3-11).

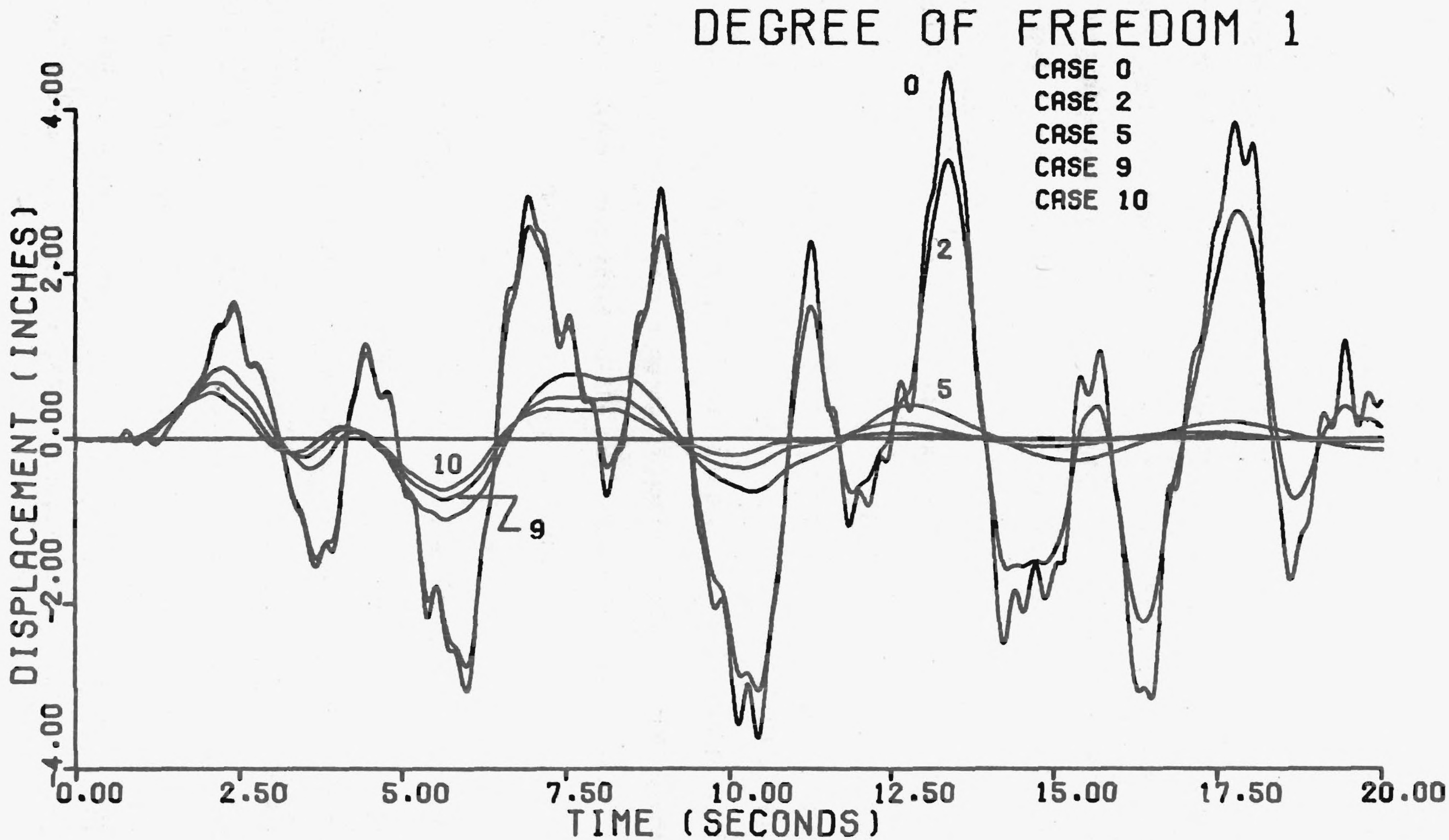



Figure 4.3-14 - Structure Response to the 10.4 second Ft. Tejon Record for Different Cases of Damping (summary of data presented in Figure 4.3-9).

and proportional damping formulations of internal damping, but proportional damping suppressed the higher modes more. This can be seen in Figures 4.3-3 and 4.3-4; the responses for cases 3 and 4 are smoother than those for cases 1 and 2.

The difference in response at degrees of freedom 1 and 28 when dampers of the same size were positioned at each level in cases 5 and 7, and at every other level in cases 6 and 8, is shown in Figures 4.3-5 through 4.3-8. Here the effect on structure response of placing dampers at every level instead of at every other level was more evident at degree of freedom 1 than at degree of freedom 28.

The effect of increasing the size (damping constant) of the add-on dampers was noticeable as can be seen in Figures 4.3-9, 4.3-10 and 4.3-14; however, increasing the damper size suppressed the free vibration response more than the response during the earthquake excitation.

The total level of damping in cases 5, 11, 12, and 13 was held constant as discussed above, but the distribution of dampers was changed in each case. Figures 4.3-11, 4.3-12 and 4.3-13 show that add-on damping was more effective when more dampers were positioned at the top of the structure than at the bottom. This was consistent with earlier observation that the structure response was concentrated in the upper more flexible part of the tower.

The maximum responses at degree of freedom 1 for the fourteen different damping cases are listed in Table 4.3-1. The largest response was 4.452 inches (0.113 m) for damping case 0, which contained zero damping. The lowest maximum response for those cases without discrete dampers was 3.008 inches (0.076 m) for case 4, which was 1% proportional damping in the two first modes. When discrete dampers were added, the maximum response decreased to 0.627 inches (0.016 m) in case 11. If cases 5, 11, 12 and 13, in which the overall level

TABLE 4.3-1. - Maximum Response at Degree of Freedom 1 and Root-Mean-Square Values for Different Damping Cases

| Damping Case[a] | RMS-Value[b] | Max-Response[b] |
|---|---|---|
| 0 | 6.553 | 4.452 |
| 1 | 5.657 | 3.835 |
| 2 | 5.088 | 3.381 |
| 3 | 5.206 | 3.520 |
| 4 | 4.686 | 3.008 |
| 5 | 2.170 | 0.986 |
| 6 | 2.554 | 1.261 |
| 7 | 2.185 | 0.993 |
| 8 | 2.590 | 1.278 |
| 9 | 1.859 | 0.745 |
| 10 | 1.705 | 0.627 |
| 11 | 2.627 | 1.399 |
| 12 | 2.000 | 0.839 |
| 13 | 1.912 | 0.741 |

[a]See Table 4.2-1.

[b]Units are inches (1 inch = 0.0254 m).

of damping was held constant, were compared, it was readily apparent that case 13 had the lowest maximum response; this was consistent with the plotted response data in Figures 4.3-11 and 4.3-12.

Response Attenuation Measures. - - Two different response attenuation measures were used to compare the overall effectiveness of the 14 different damping cases. In the first, the root-mean-square (RMS) values of the maximum responses at the master degrees of freedom for damping cases 0 to 13 were computed and are listed in Table 4.3-1. In general the RMS and maximum responses both decrease for increasing levels of overall damping.

As a second measure of damper effectiveness, the logarithmic decrement [65] , which indicates the level of decay of free vibration response, was also used to compare structure response for the different damping cases. The logarithmic decrement was taken to be a convenient measure of the overall damping effect of both internal and add-on damping, and was used because the structure was vibrating primarily in the first mode during free vibration response. The equivalent, first mode damping ratios for cases 5 through 13 were computed by dividing the logarithmic decrement by $2\pi$ [65] and are listed in Table 4.3-2. Case 10 produced the highest damping ratio of 21.7% while case 8 yielded the lowest, 6.5%. When the cases of equal overall damping (5, 11, 12, and 13) were compared, the highest damping ratio (10.5%) was observed for case 5 and the second highest (10%) for case 12. An interesting observation was made by comparing Tables 4.3-1 and 4.3-2. Maximum response at degree of freedom 1 was lower for cases 12 and 13 than for case 5, but the overall damping ratio $\gamma$ was higher for case 5 than cases 12 and 13. Therefore, it was concluded that placing more dampers at the top of the prototype structure reduced the maximum and RMS seismic responses more than a uniform distribution of dampers of equal size at every level. However, the rate of decay of free vibration response

TABLE 4.3-2. - Equivalent First Mode Damping Ratio for Response at Degree of Freedom 1 to the Ft. Tejon Record and Different Damping Cases

| Damping Case[a] | Equivalent First Mode Damping Ratio $\gamma$, in percent of critical viscous damping |
|---|---|
| 5 | 10.5 |
| 6 | 7.1 |
| 7 | 10.0 |
| 8 | 6.5 |
| 9 | 14.0 |
| 10 | 21.7 |
| 11 | 7.6 |
| 12 | 10.0 |
| 13 | 8.1 |

[a]See Table 4.2-1.

measured by damping ratio $\gamma$, was greater for the uniform distribution case.

## 4.4 Conclusions

A study of the effect of damping on the dynamic response of freestanding tower structures has been initiated. Evaluating the effects of internal damping using either the simple modal or proportional damping formulation produced results that were reasonable and expected. It was shown, for example, that the proportional damping formulation did suppress the higher modes more quickly, as expected. Varying the strength and distribution of add-on dampers produced noticeable differences in the level of x direction dynamic response.

In general the results obtained in this study lead to the following conclusions:

(1) The effectiveness of damping during the earthquake excitation was small but in the free vibration portion of the response, damping did influence response.

(2) The simple modal and proportional damping formulations of internal damping had the same overall effect on the dynamic response of the prototype structure.

(3) While the addition of discrete damping devices did not appreciably alter the character of the response, the response level was noticeably affected.

(4) The damper size had a significant influence on structure response, but the effectiveness of the damper decreases with increasing size.

(5) For the structure, loading, and damping cases considered, the use of 20 instead of 10 dampers of uniform size and distribution had little effect in further reducing the dynamic response.

(6) Maintaining the total damping force constant and varying the distribution of the size of the dampers demonstrated that the distribution of dampers had a major influence on the dynamic response of the prototype structure.

More studies involving a wider range of loadings are needed before general conclusions can be made about the optimum size or number of dampers to be used to limit dynamic response of the prototype structure. However, preliminary findings indicate that the size and distribution of dampers had relatively more influence on tower response than the number of dampers.

# Chapter 5
# NONLINEAR RESPONSE STUDIES

## 5.1 Introduction

Slender cable-like members, referred to as tension-only members in this report, are frequently used in communication towers to stabilize the structure and to carry tension forces. However, the slender members pose a problem to the structural analyst in that they are incapable of carrying compression forces unless some initial pretensioning is applied. The analyst may choose to model the tension-only members in an approximate manner as ordinary tension/compression members with actual or reduced cross-sectional areas, or to consider the true nonlinear behavior of these members if their contribution to the overall performance of the structure is felt to be significant.

An additional consideration is that the contribution of tension-only members to overall structure stiffness is dependent upon several other factors, namely (1) the number and arrangement of tension-only members, (2) the magnitude and direction of loading, and (3) the level of initial pretensioning applied to the tension-only members. In the case of dynamic loading, the force level in the tension-only members varies with time and may result in selected members becoming ineffective for certain periods of time during the dynamic response of the structure.

In this chapter, a procedure for including tension-only member nonlinearity in the static and dynamic response analysis of tower structures is presented. The nonlinear behavior of tension-only members was represented by the stress-strain curve in Figure 5.1-1. For tension forces, a linear elastic stress-strain relation was used but for compression, a curve with zero slope was specified. Results of nonlinear and approximate linear analyses for a section of the prototype structure were compared, and the importance of including the nonlinear effect was investigated for several static and dynamic loadings.

Objectives. - - Since nonlinear static and dynamic analyses are much more difficult to perform than linear analyses, it would be convenient to be able to ignore the nonlinear behavior of tension-only members in the response analysis of tower structures. As an approximation, it was initially assumed in this study (see discussion in Chapter 3) that the actual nonlinear structural behavior could be bracketed by two linear analyses. In both cases, tension-only members were assumed to be capable of carrying both tension and compression forces, but, in the first case, the actual cross-sectional areas

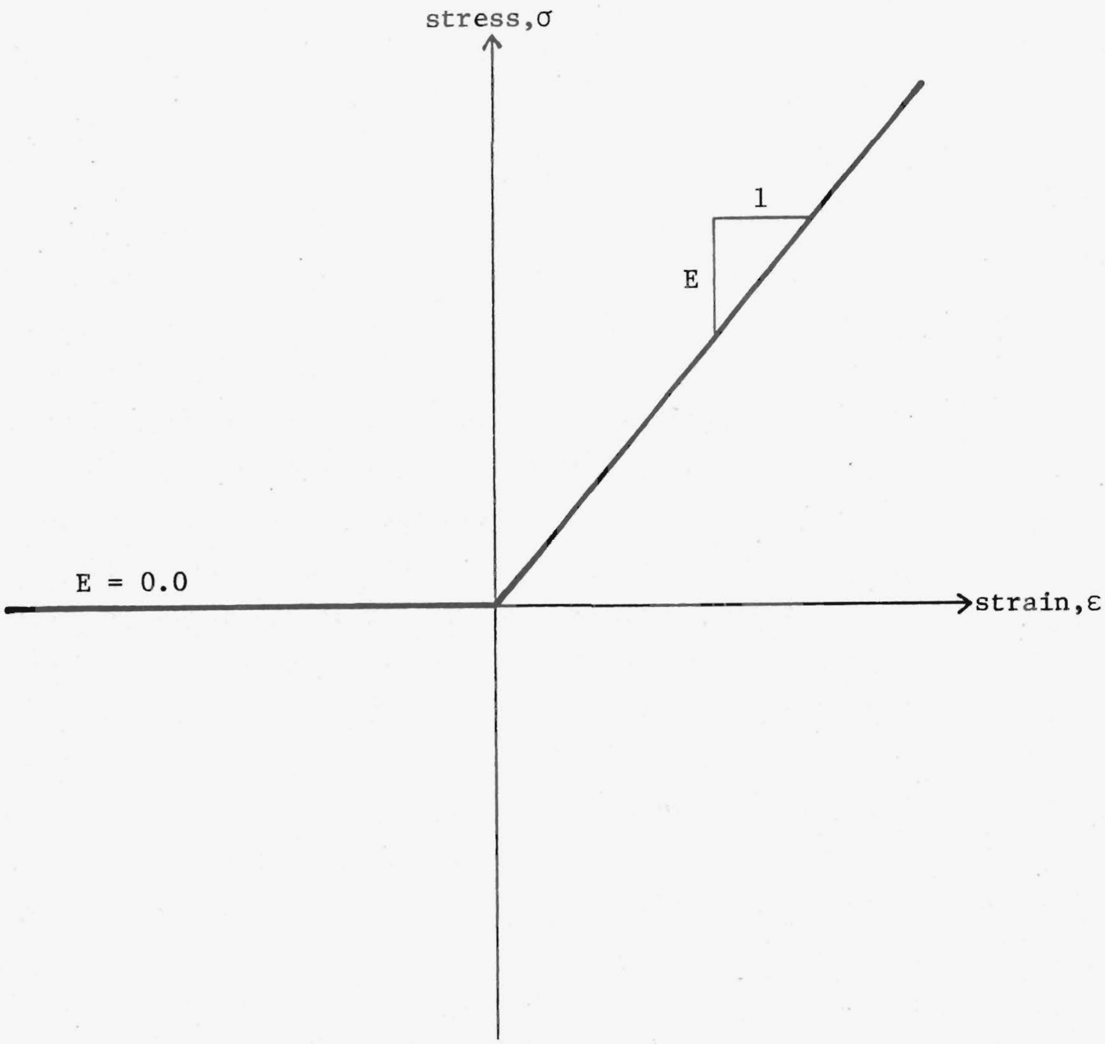



Figure 5.1-1 - Tension-only Member Stress-Strain Curve

of these members were used, and, in the second case, very small areas were assigned to these members. It was not possible to reduce the cross-sectional areas to zero since the example structures studied became unstable for this case. Finally, the actual nonlinear response was computed and compared to the approximate linear results.

The principal objectives of the studies were (1) to investigate the use of nonlinear solution techniques to model tension-only member behavior, (2) to determine the influence of loading characteristics, damping, and pretensioning on structure response, and (3) to study the degree of approximation involved in using the linear solution procedures for the static and dynamic analyses of a portion of the prototype structure containing a number of tension-only members.

Scope. - - The iterative initial stress method was used to determine the static and dynamic response of space truss structures including the nonlinear tension-only member effect. In the case of dynamic response analysis, the iteration procedure was combined with the direct linear extrapolation technique presented in Chapter 4 to form a step-iterative procedure. Both the iterative and step-iterative procedures are described below in Section 5.2.

The static and dynamic response of a section of the prototype structure, described in Section 5.3, was computed for several loadings, and the difference in response resulting from use of actual nonlinear and approximate linear models for tension-only members was compared. The influence of loading characteristics, damping, and initial pretensioning of tension-only members were studied, and parameter study results are presented in Section 5.4. Finally, conclusions based upon the nonlinear studies are presented in Section 5.5.

Other than the material nonlinearity associated with the tension-only member behavior, the response of the space truss structure model was assumed to be both geometrically and materially linear.

## 5.2 Initial Stress Method

Initially, two nonlinear solution procedures were considered for study of the tension-only member problem. In the initial strain method, strains are determined in terms of stresses so this approach could not be used. From Figure 5.1-1 it is evident that no unique level of strain exists in a tension-only member for zero stress. For the initial stress method, stresses are determined in terms of strains, and for the tension-only member problem a unique level of stress does exist for every level of strain. Details of the method and the solution procedures employed are presented below.

Solution Procedures. - - A finite element solution of a nonlinear material problem is usually obtained using one of the three basic techniques: incremental or stepwise procedures, iterative or Newton methods, and step-iterative or mixed procedures [15,50].

In the incremental or stepwise procedure, the load on a structure is divided into many small partial loads or increments. The load increments are applied one at a time, and during the application of each increment the equations are assumed to be linear. The incremental procedure approximates a nonlinear problem as a series of linear problems, or as piecewise-linear.

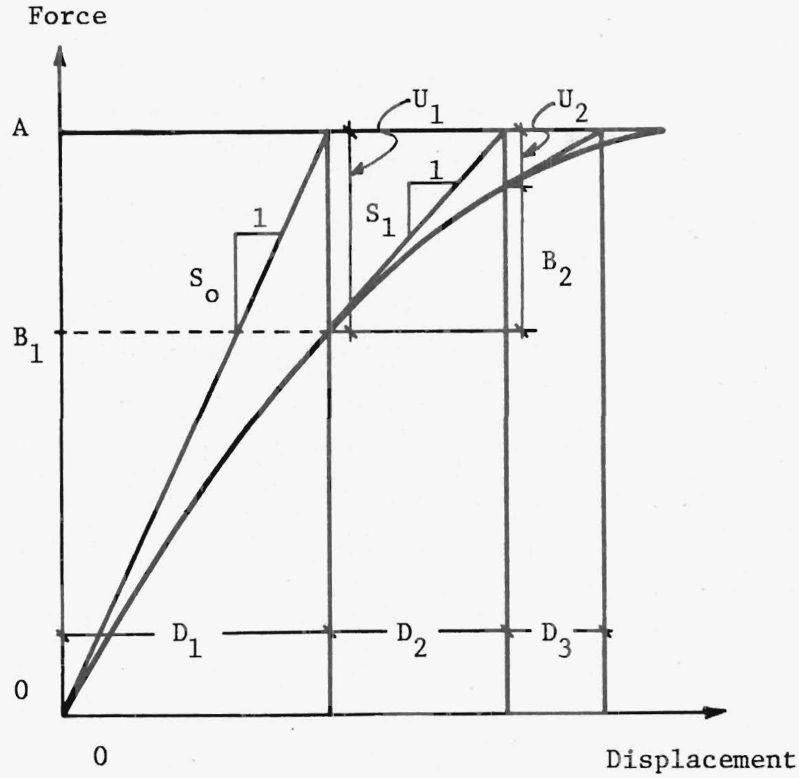
An iterative method is much simpler to program than an incremental procedure, and is more computationally efficient. However, for certain problems, iterative methods have serious disadvantages compared to incremental solution procedures. The iterative techniques cannot be employed for path-dependent or hysteretic problems, and iterative solution procedures are generally not applicable to dynamics problems. The tension-only member problem is not path-dependent, and a step-iterative approach was used for dynamic analysis.

In the iterative or Newton method, the structure is fully loaded in the initial cycle as illustrated in Figure 5.2-1(a). To start the procedure, the structure response is found, by assuming linear behavior, from the equation

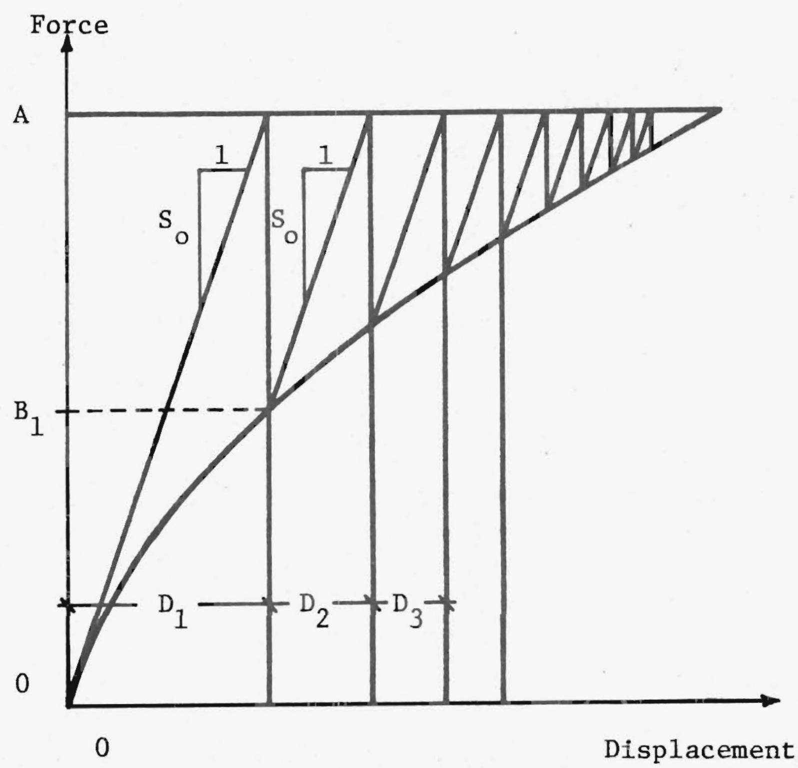
$$\tilde{D}_1 = \tilde{S}_o^{-1} \tilde{A} \tag{5.2-1}$$

where $\tilde{D}_1$ is the displacement vector for cycle 1, $\tilde{A}$ is the total load vector, and $\tilde{S}_o$ is the initial structure stiffness matrix. Because a constant value of stiffness is used for cycle 1, and some tension-only members may become ineffective as a result of the loading, not all of load $\tilde{A}$ is balanced and the unbalanced load $\tilde{U}_1$ must be reapplied in cycle 2.

The elongation $\Delta$ of the structural members can be found from the displacements $\tilde{D}_1$. The strain in member i, $\varepsilon_i$, can then be calculated as




(a) Variable Stiffness Iterative Procedure




(b) Constant Stiffness Iterative Procedure

Figure 5.2-1 - Iterative Procedures for Nonlinear Static Analysis

$$\varepsilon_i = \frac{\Delta_i}{\ell_i} \tag{5.2-2}$$

where $\ell_i$ is the length of truss member i. With the appropriate stress-strain relationship specified for both regular and tension-only members, the stress and force in member i can be obtained from $\varepsilon_i$. For compression strain, the stress in tension-only members is set to zero. Summing member forces acting at each node, a nodal force vector equal to but opposite in sign to the balanced load vector, $\underset{\sim}{B}_1$ is obtained. The unbalanced load $\underset{\sim}{U}_1$, which must be applied to the structure in cycle 2, is equal to

$$\underset{\sim}{U}_1 = \underset{\sim}{A} - \underset{\sim}{B}_1 \tag{5.2-3}$$

The additional structure displacements $\underset{\sim}{D}_2$ are obtained from the equation

$$\underset{\sim}{D}_2 = \underset{\sim}{S}_1^{-1} \underset{\sim}{U}_1 \tag{5.2-4}$$

where $\underset{\sim}{S}_1$ is the tangent stiffness calculated at the end of cycle 1. Again, in cycle 2, not all of load $\underset{\sim}{U}_1$ is balanced and the process is repeated to obtain the unbalanced load $\underset{\sim}{U}_2$ to be applied in cycle 3. In general, the unbalanced load from cycle j is

$$\underset{\sim}{U}_j = \underset{\sim}{A} - \sum_{k=1}^{j} \underset{\sim}{B}_k \tag{5.2-5}$$

and $\underset{\sim}{U}_j$ is applied as the structure loading for cycle j + 1. The additional displacements resulting from application of joint loads $\underset{\sim}{U}_j$ in cycle j + 1 are

$$\underset{\sim}{D}_{j+1} = \underset{\sim}{S}_j^{-1} \underset{\sim}{U}_j \tag{5.2-6}$$

The iterative procedure ends when the RMS value of the unbalanced load vector $\underset{\sim}{U}_n$ for cycle n is less that some specified tolerance. Total structure response,

$\underset{\sim}{D}$, is expressed as

$$\underset{\sim}{D} = \sum_{j=1}^{n} \underset{\sim}{D}_j \tag{5.2-7}$$

Only total displacement is found using the iterative procedure, while a load-displacement history is obtained from the incremental method.

A variation of the iterative method is to continue to use the initial structure stiffness $\underset{\sim}{S}_o$ throughout the iteration procedure as shown in Figure 5.2-1(b). This approach requires more iterations than the basic procedure, but often results in fewer total calculations because the structure stiffness matrix does not have to be reformed and inverted for each cycle. The constant stiffness approach was used in this investigation.

Finally the step-iterative procedure represented in Figure 5.2-2 combines the incremental and iterative solution approaches to yield the response path of the structure. The load on the structure is applied incrementally ($\Delta A_i$ for step i in Figure 5.2-2) and the iterative method is used to successively correct the results within the step until equilibrium is achieved. The incremental displacement response for step i $\Delta D_i$ is added to the total displacement $D_{i-1}$ at the end of the previous step to obtain response $D_i$.

In a dynamic response analysis, both the load and displacement are functions of time. The response analysis consists of solving a series of equivalent static problems at intervals of time $\Delta t$, as discussed in Chapter 4 in connection with the direct linear extrapolation procedure. Now, iterations must be used within each time step to reduce the unbalanced dynamic load in each step to a negligible value. The step-iterative procedure used for dynamic response computations is presented below.

Iterative Initial Stress Procedure for Static Analysis. - - The iteration procedure used to compute the total structure displacements, without consideration of the effect of initial pretensioning of tension-only members, can be summarized as follows:

1. Compute the structure stiffness matrix $\underset{\sim}{S}_o$ assuming that tension-only members are effective in both tension and compression;
2. Form the joint force vector $\underset{\sim}{A}$ and compute the initial displacement vector $\underset{\sim}{D}_1$ from Equation (5.2-1);

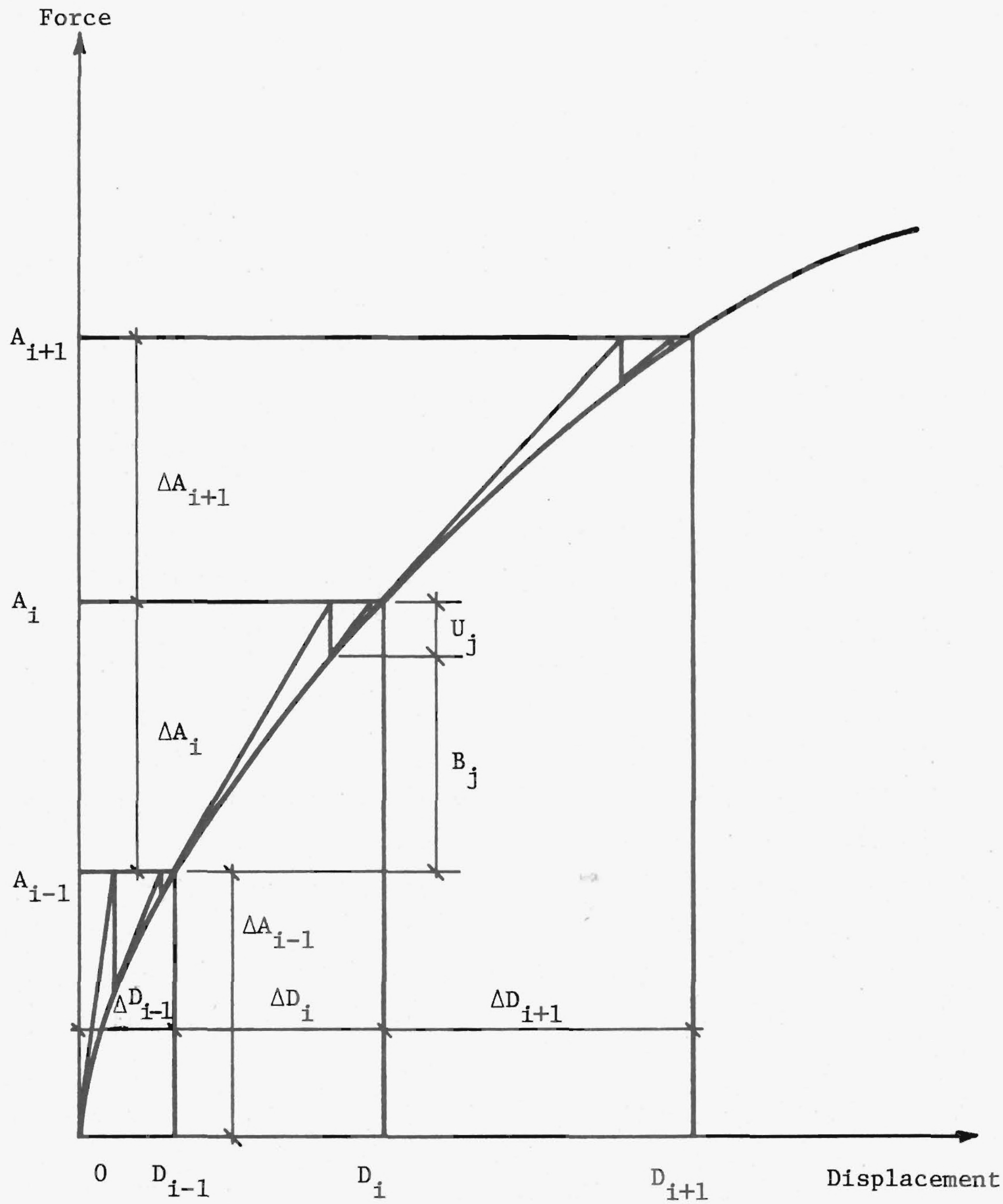



Figure 5.2-2 - General Step-Iterative Procedure for Nonlinear Analysis

3. Compute member strains $\varepsilon_1$ from Equation (5.2-2) using the displacement vector $\underset{\sim}{D}_j$ for the current iteration cycle j, then compute member forces; if a tension-only member experiences compressive strain, reduce its member force to zero;
4. Calculate the unbalanced load vector $\underset{\sim}{U}_j$, whose entries are to be applied as joint loads in the next cycle of iteration, from Equation (5.2-5); check that the RMS value of $\underset{\sim}{U}_j$ is less than the specified tolerance;
5. If the RMS of $\underset{\sim}{U}_j$ exceeds the acceptable tolerance value, compute $\underset{\sim}{D}_{j+1}$ for the next cycle from Equation (5.2-6), then go to step 3 above; if the RMS of $\underset{\sim}{U}_j$, is sufficiently small go to step 6;
6. Compute the total structure displacements $\underset{\sim}{D}$ from Equation (5.2-7).

In many instances tension-only members are initially stressed in tension in a structure during erection, before external load is applied. In this way, if compression forces develop in tension-only members when actual load is applied, the initial tension forces will act to balance the compression forces. The above procedure must be modified slightly if the pretension effect is to be included in the analysis.

It must first be recognized that pretensioning causes initial forces in other members of the structure. The assumption was made for this study that pretensioned members act together and influence other members of the structure, but that the effect they have on one another is included in the specified pretensioning force. Because not all tension-only members may be pretensioned in a structure, some tension-only members may be compressed by the initial pretensioning forces in the other tension-only members. Therefore, to find all initial member forces in the structure prior to the application of external loading, the iterative initial stress procedure outlined above was employed. In the analysis, pretensioned members were removed from the structure and the specified pretension forces were applied to the structure as equivalent joint loads. With this analysis complete, the pretensioned members were reinserted in the structure, the external joint force vector $\underset{\sim}{A}$ in step 2 was formed, and the iterative initial stress procedure was applied in the form presented above. In step 3, the cumulative effects of initial pretensioning and member forces caused by external loading were added to determine the current status of all tension-only members.

Step-Iterative Initial Stress Procedure for Dynamic Analysis. - - A direct integration procedure, employing linear extrapolation and the trapezoidal rule, was described in Chapter 4 and was used for linear dynamic analysis of the prototype structure. Now, the principal steps in the step-iterative procedure used for nonlinear dynamic analysis will be presented. The procedure is illustrated in Figure 5.2-3.

The principal steps are:

1. First compute initial forces in all members of the structure due to any specified pretension forces, using the static analysis iteration procedure.
2. Form the stiffness matrix $S^*$ using Equation 4.3-9, substituting $S_o$ from the iteration procedure in place of $S$; since constant stiffness iteration was used throughout, refer to matrix $S^*$ as $S_o^*$ in what follows.
3. Calculate the pseudo-static load vector $A_i^*$ (Equation 4.3-10) for step i using displacements $D_{i-1}$, obtained by iteration in step i-1, and $\dot{D}_{i-1}$ and $\ddot{D}_{i-1}$ as initial conditions for step i.
4. To start the iteration procedure for dynamic analysis, solve for the initial estimate of $D_i$ (referred to as $D_i^e$ in Figure 5.2-3) using Equation (4.3-8).
5. Check for net compressive forces in tension-only members and reduce forces to zero if a net compression is detected; compute the balanced load vector $B_j^i$ (Figure 5.2-3) for the jth iteration in step i from the equivalent nodal forces associated with the adjusted member forces; finally, compute the unbalanced load vector $U_j^i$ resulting from the previous iteration in step i as

$$U_j^i = S_o D_i - B_j^i \qquad (5.2\text{-}8)$$

   If the RMS of $U_j^i$ is sufficiently small, go to step 9 below.
6. Apply $U_j^i$ as joint loads to the structure and compute displacements $\Delta D_j^i$ for iteration j within step i as

$$\Delta D_j^i = (S_o^*)^{-1}\, U_j^i \qquad (5.2\text{-}9)$$

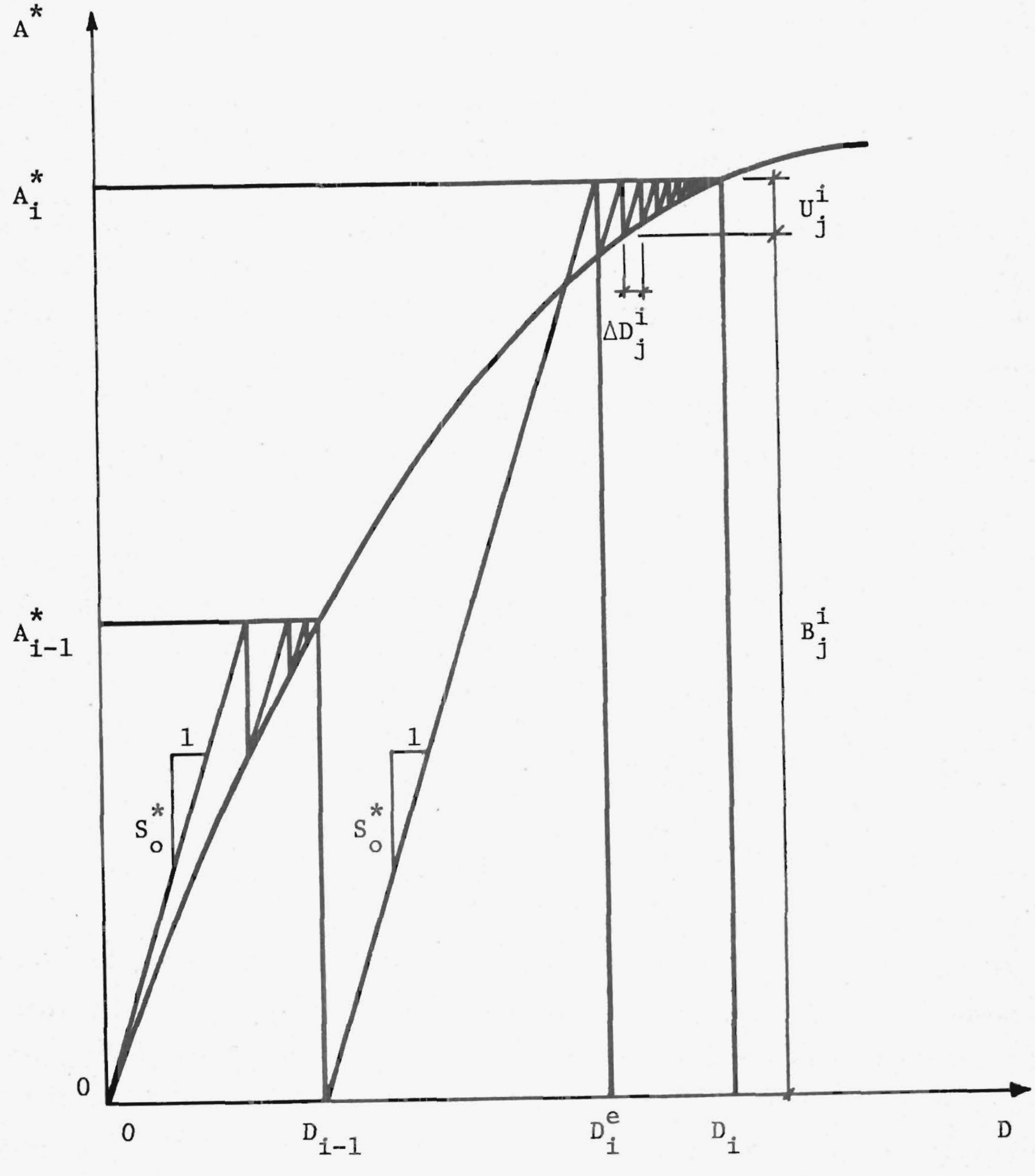



Figure 5.2-3 - Iteration Combined with Direct Linear Extrapolation for Nonlinear Dynamic Analysis

7. Compute the current estimate of $\underset{\sim}{D}_i$ as

$$\underset{\sim}{D}_i = \underset{\sim}{D}_i^e + \sum_{k=1}^{j} \Delta \underset{\sim}{D}_k^i \qquad (5.2\text{-}10)$$

8. Repeat steps 5, 6, and 7 until the RMS of $\underset{\sim}{U}_j^i$ is sufficiently small.
9. Calculate $\dot{\underset{\sim}{D}}_i$ and $\ddot{\underset{\sim}{D}}_i$ from Equations (4.3-6) and (4.3-7), using $\underset{\sim}{D}_i$ from Equation (5.2-10).
10. Let step i become step i-1 and return to step 3 to begin the calculations for the next time increment. Terminate the procedure when response for the last time increment has been obtained.

Computer Program. - - A FORTRAN computer program was written to implement the above nonlinear solution techniques for static and dynamic analysis of space truss structures with tension-only members. This program was used to study one section of the large tower structure for several static and dynamic loading cases to determine the importance of including the nonlinear tension-only effect in the analysis of the prototype structure; results are presented below.

## 5.3 Example Problem

Section ZB of the prototype structure (see Appendix A) was selected for detailed study of the tension-only member effect. This segment of the large tower contained 12 tension-only members and was considered to be a typical section on the tower. The size of the large tower prohibited computer analyses of the entire structure which included the nonlinear tension-only effect. The influence of tension-only members on the behavior of the single section was assumed to be representative of that which would be obtained if analyses of the entire tower were performed.

The model of the tower section shown in Figure 5.3-1 contains 48 joints and 135 members. Fifteen of the 135 members were added to stabilize planar joints. The stabilizing members were positioned normal to the plane of the planar joints and computed member forces were zero for these members. The tower model contains 3 degrees-of-freedom per joint, tributary masses were lumped at the joints for dynamic analyses, and pin supports are indicated in the figure.

Two static and two dynamic loadings were applied to corner joints 34, 38, and 42 at the top of the section. The static loadings (Figure 5.3-2) consist

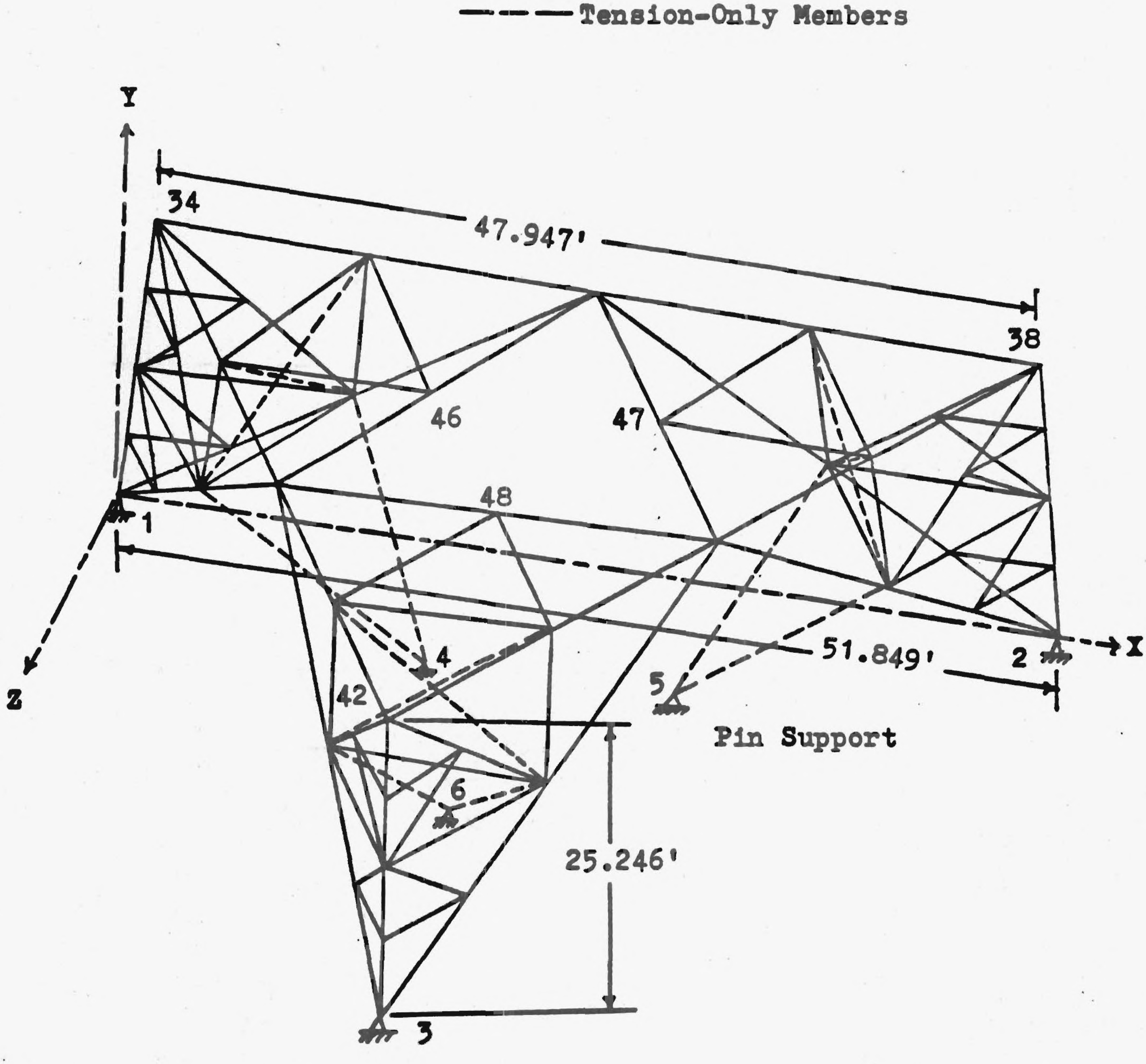
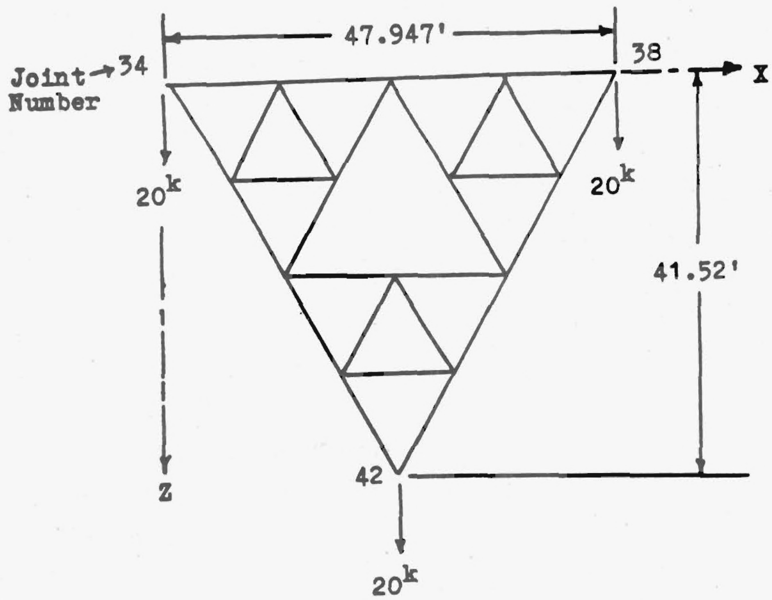
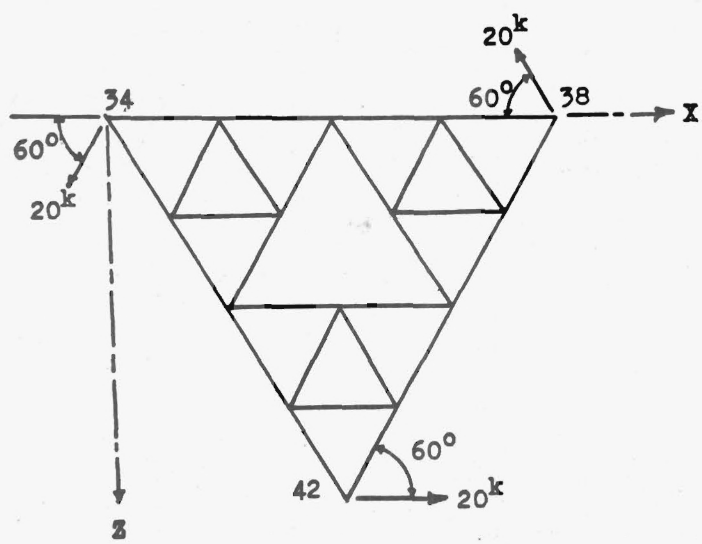



Figure 5.3-1 - Section ZB of the Large Tower (1 ft = 0.3048 m)




(a) Loading Case 1




(b) Loading Case 2

Figure 5.3-2 - Static Loadings for Section ZB of the Large Tower
(Note: 1 ft = 0.3048 m, 1 kip = 4.45 kN)

of three 20 kip (89 kN) loads yielding translational and torsional response. The dynamic loadings (Figure 5.3-3) were arranged in the same manner as the static loads, but were applied as step functions with a magnitude of 10 kips (44.5 kN).

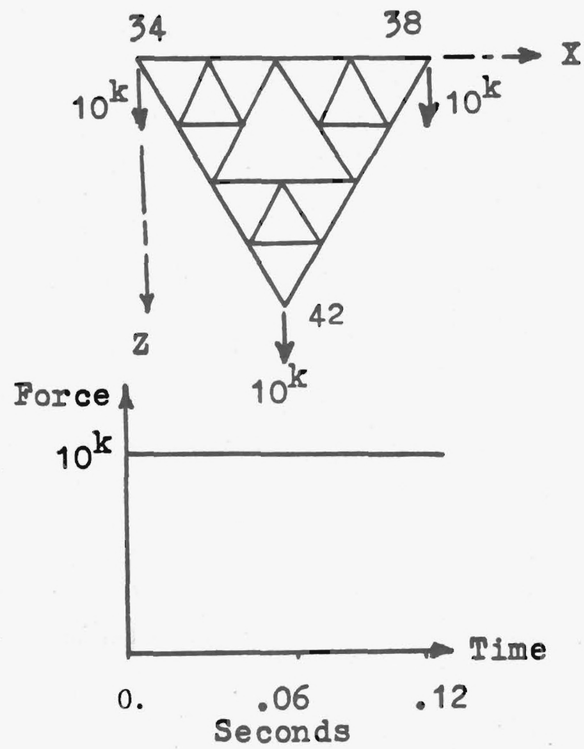
Proportional damping was used for the dynamic response studies of Section ZB to be presented below. In each case, damping ratios were specified for modes 1 and 2 of the model, and used to determine the two constants in the proportional damping formulation [65].

## 5.4 Parameter Studies

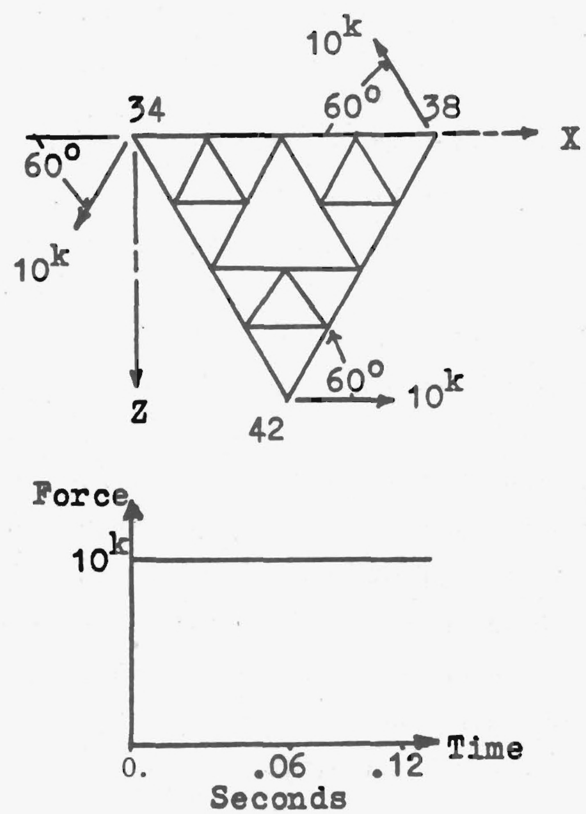
Static Analyses. - - The displacement response of joint 42 at the top of the model (Figure 5.3-1) to static loadings 1 and 2 (Figure 5.3-2) is listed in Table 5.4-1. Only displacements in the direction of loading are reported for each case. A variety of structure modifications and analysis types were considered: (1) linear analysis with tension-only members treated as tension-compression elements having cross-sectional areas of 0.307 $in^2$ (1.98 $cm^2$); (2) linear analysis with reduced cross-sectional areas of 0.005 $in^2$ (0.0322 $cm^2$) for tension-only members; (3) nonlinear analysis with no initial pretensioning of tension-only members; and (4) nonlinear analysis with varying pretension force levels.

The smallest response results in Table 5.4-1 were obtained from the linear analysis with original tension-only member cross-sectional areas of 0.307 $in^2$ (1.98 $cm^2$) specified. Larger joint displacement and member stress values were obtained using the nonlinear solution procedure with tension-only members allowed to develop only tension forces. The largest response results were found using the linear solution procedure with tension-only member cross-sectional areas reduced to 0.005 $in^2$ (0.0322 $cm^2$). A 5% variation in response results was observed for the three solution procedures for loading case 1, and a 3% difference for loading case 2.

The Parameter study results which include the effects of pretensioning on the static response of the model to loadings 1 and 2 are also reported in Table 5.4-1. Only six tension-only members were significantly stressed by loading 1, so only these six members were pretensioned initially. The six tension-only members were those incident on joints 4, 5, and 6 (Figure 5.3-1), which acted as interior guy wires for the model. Next all twelve tension-only members were pretensioned, and the structure response again




(a) Loading Case 1




(b) Loading Case 2

Figure 5.3-3 - Dynamic Loading Cases for Section ZB of the Large Tower

Table 5.4-1 - Displacement Response of Joint 42 of Section ZB of the Prototype Structure to Static Loading Cases 1 and 2, in feet[a]

| Type of Analysis (1) | Tension-only Member Area, in square inches[b] (2) | Pretension Force, in kips[c] (3) | Z-direction Response to Loading 1, in feet[a] (x $10^{-2}$) (4) | X-direction Response to Loading 2, in feet[a] (x $10^{-2}$) (5) |
|---|---|---|---|---|
| Linear | 0.307 | 0. | 1.326 | 3.318 |
| Linear | 0.005 | 0. | 1.540 | 3.417 |
| Nonlinear | 0.307 | 0. | 1.365 | 3.364 |
| Nonlinear | 0.307 | 3.0[d] | 1.352 | 3.347 |
| Nonlinear | 0.307 | 3.0[e] | 1.350 | 3.340 |
| Nonlinear | 0.307 | 5.0[d] | 1.336 | 3.340 |
| Nonlinear | 0.307 | 5.0[e] | 1.334 | 3.322 |
| Nonlinear | 0.307 | 10.0[d] | 1.327 | 3.325 |
| Nonlinear | 0.307 | 10.0[e] | 1.326 | 3.318 |

[a]Note: 1 ft = 0.3048 m

[b]Note: 1 $in^2$ = 645.2 $mm^2$

[c]Note: 1 kip = 4.448 kN

[d]Tension-only members incident upon joints 4, 5, and 6 (Figure 5.3-1) only.

[e]All twelve tension-only members.

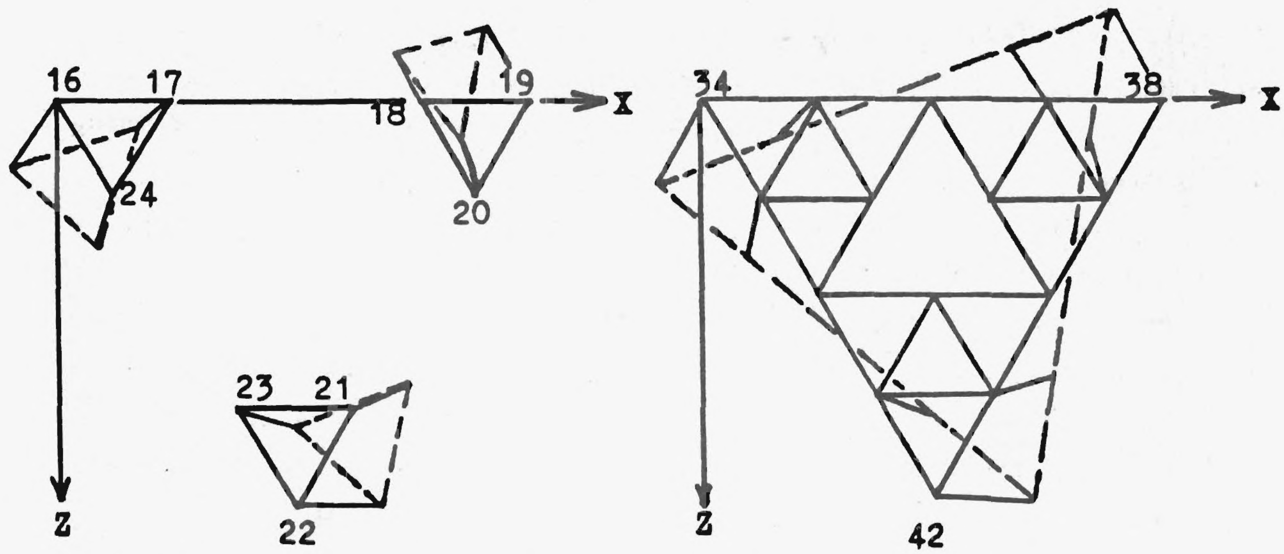
determined. Pretensioning six members affected response results almost as much as pretensioning all twelve tension-only members. By increasing the pretensioning forces displacement results were reduced to the minimum value obtained from the linear solution for structure response with original tension-only member cross-sectional areas specified and no pretensioning. The minimum displacement results were obtained when pretensioning forces were greater than compression forces caused in tension-only members by external loads; for loading case 1 a force of 10 kips (44.48 kN) was required. However, pretensioning also increased member forces in the structure.

All twelve tension-only members were stressed by loading case 2. Therefore, the difference in displacement results obtained by pretensioning six or twelve tension-only members was larger for loading case 2 than for loading case 1. Increasing the pretensioning force reduced structure displacement response. The minimum displacements were found to occur when the pretensioning forces in tension-only members were greater than compression forces caused by external loads. The minimum displacements were the same as the linear analysis results with original tension-only member cross-sectional areas specified. Member forces were again increased by pretensioning.

Dynamic Analysis. - - The step function loadings (cases 1 and 2) in Figure 5.3-3 were applied to joints 34, 38, and 42 in Section ZB (Figure 5.3-1) of the prototype structure to study the influence of tension-only members on dynamic response of the model. Results are presented in the form of displacement-time history plots for selected degrees-of-freedom in the model. Variations in response plots illustrate the difference between linear and nonlinear solution procedures.

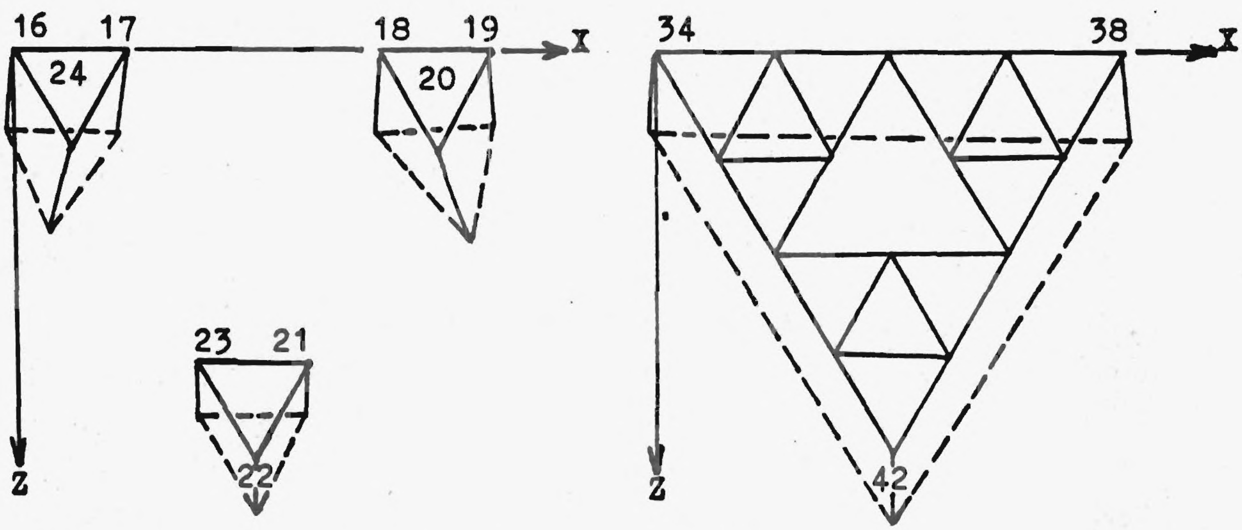
Initially, vibration periods and mode shapes for the lower modes were determined using GTSTRUDL. Modes 1 and 2 are displayed in Figure 5.4-1. The corresponding periods are 0.060 seconds and 0.058 seconds respectively, and the period of the highest significant mode was determined to be 0.020 seconds. Therefore, a uniform time increment of 0.002 seconds was used for all dynamic response computations in the numerical integration procedure.

As in the static analyses described above, four dynamic analysis cases were considered: (1) and (2), linear analysis with actual ($0.307\ in^2 = 1.98\ cm^2$) and reduced ($0.001\ in^2 = 0.0065\ cm^2$) cross-sectional areas (Ax) for tension-only members treated as ordinary tension-compression elements; (3) nonlinear analysis without pretension forces; and (4) nonlinear analysis




Mid-Height Cross Section Top Cross Section

(a) First Mode Shape, Period = 0.06 Seconds




Mid-Height Cross Section Top Cross Section

(b) Second Mode Shape, Period = 0.058 Seconds

Figure 5.4-1 - Plan View of Mode Shapes for Modes One and Two of Section ZB of the Prototype Structure with specified pretension forces in tension-only members. Damping was neglected initially; later, 1% critical viscous damping was specified in modes 1 and 2 to assemble a proportional damping matrix, and damping and tension-only effects on model response were compared.

Z-diréction response at joint 42 for loading 1 and X-direction response at joint 42 for loading 2 are compared in Figures 5.4-2 and 5.4-3, respectively, for the linear and nonlinear cases described above. The linear results bracket the nonlinear results for loading 1 (Figure 5.4-2), and the three solutions vary by as much as 6%. The linear case in which actual cross-sectional areas ($A_x$ = 0.307 in$^2$, 1.98 cm$^2$) of tension-only members were used provides a lower bound, and the linear case with reduced cross-sectional areas ($A_x$ = 0.001 in$^2$, 0.0065 cm$^2$) yields an upper bound to the displacement response. This was not true of the response to loading 2, however, in which higher mode response is evident (Figure 5.4-3). Accounting for the true nonlinear behavior of tension-only members resulted in larger displacements for some time increments. However, the maximum displacements, considering all time steps, were obtained using the linear solution procedure with reduced tension-only member cross-sectional areas. Linear and nonlinear results varied by as much as 8% for loading 2.

Next, pretension forces of 10 kips (44.5 kN) were specified for all tension-only members in the model and nonlinear dynamic analyses performed for loading cases 1 and 2. Actual cross-sectional areas of tension-only members were used. The time-history responses are compared to results obtained from nonlinear analyses without the effect of pretensioning in Figures 5.4-4 and 5.4-5 for loadings 1 and 2, respectively. The 10 kip (44.5 kN) pretension forces turned out to be greater than all compression forces developed in tension-only members as a result of the loadings. Therefore, net tension forces were maintained in all tension-only members for the duration of both loadings 1 and 2. As a result, the nonlinear analysis results including initial pretensioning displayed in Figures 5.4-4 and 5.4-5 are equivalent to the linear analysis results in which actual cross-sectional areas were used in Figures 5.4-2 and 5.4-3 respectively.

The final parameter considered was damping. As noted above, 1% of critical viscous damping was specified in modes 1 and 2 to assemble a

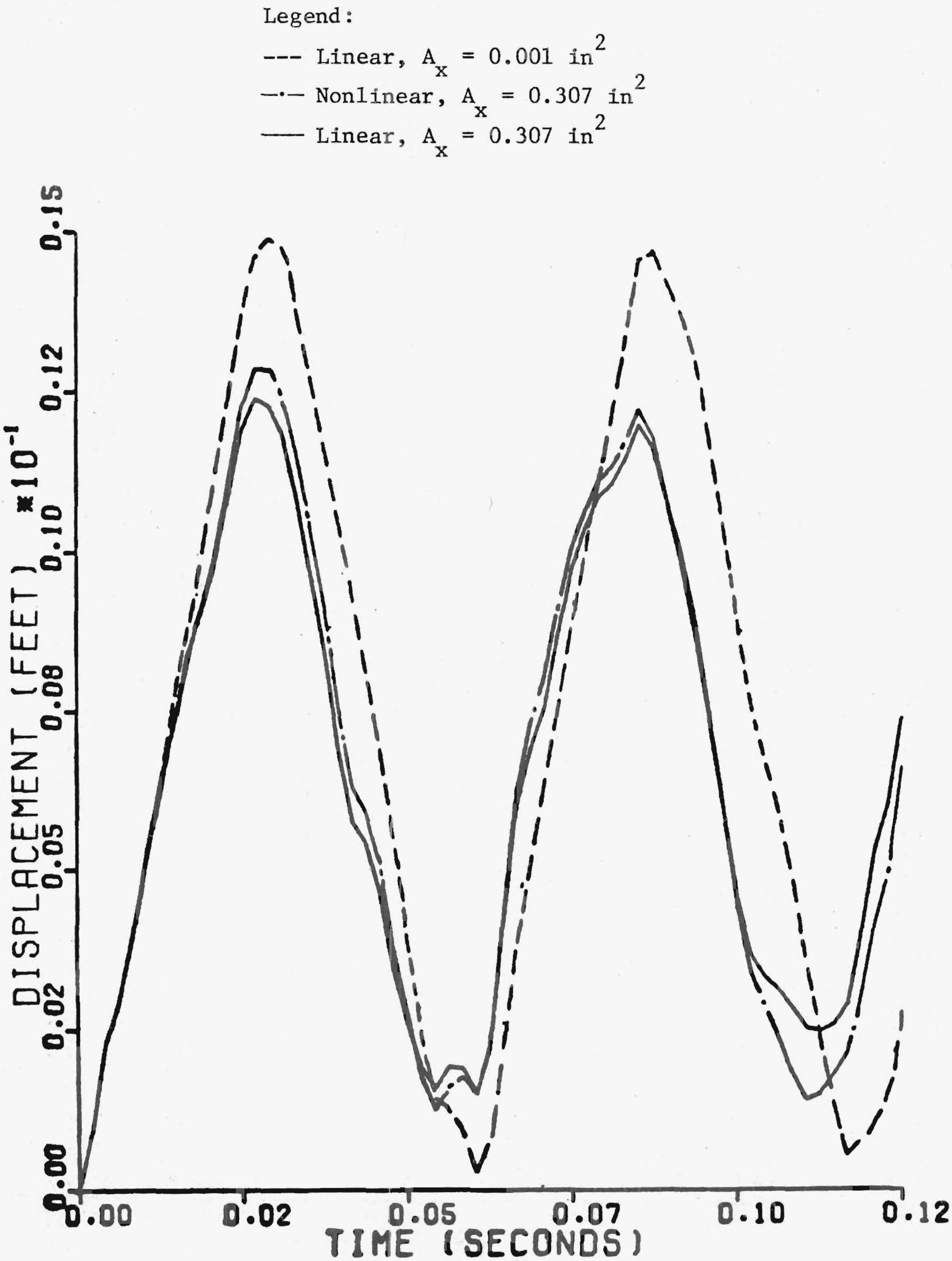



Figure 5.4-2 - Z-direction Undamped Dynamic Response at Joint 42 to Loading 1 for Varying Cross-Sectional Areas of Tension-only Members, No Pretensioning Applied.

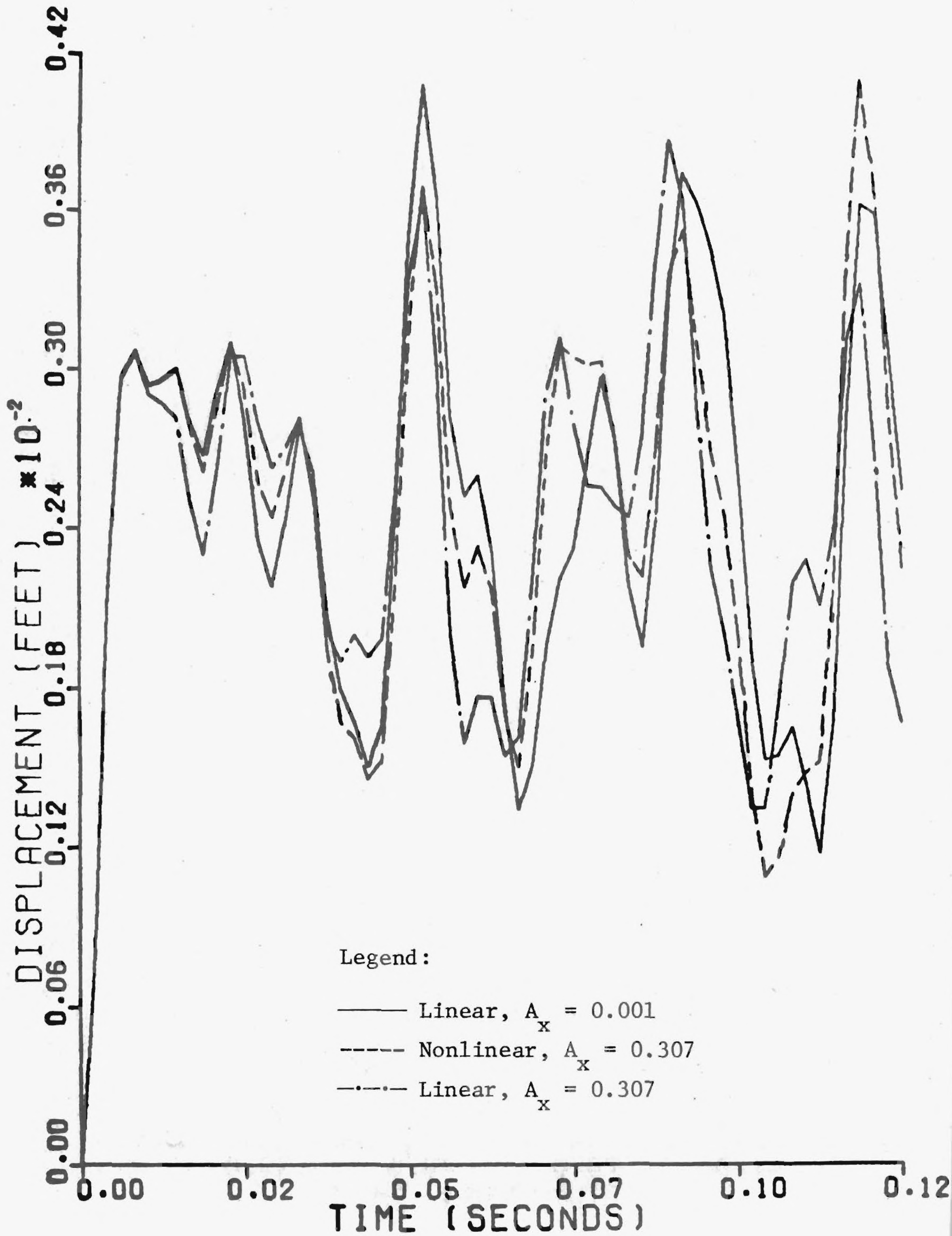



Figure 5.4-3 - X-direction Undamped Dynamic Response at Joint 42 Loading 2 for Varying Cross-Sectional Areas of Te only Members, No Pretensioning Applied.

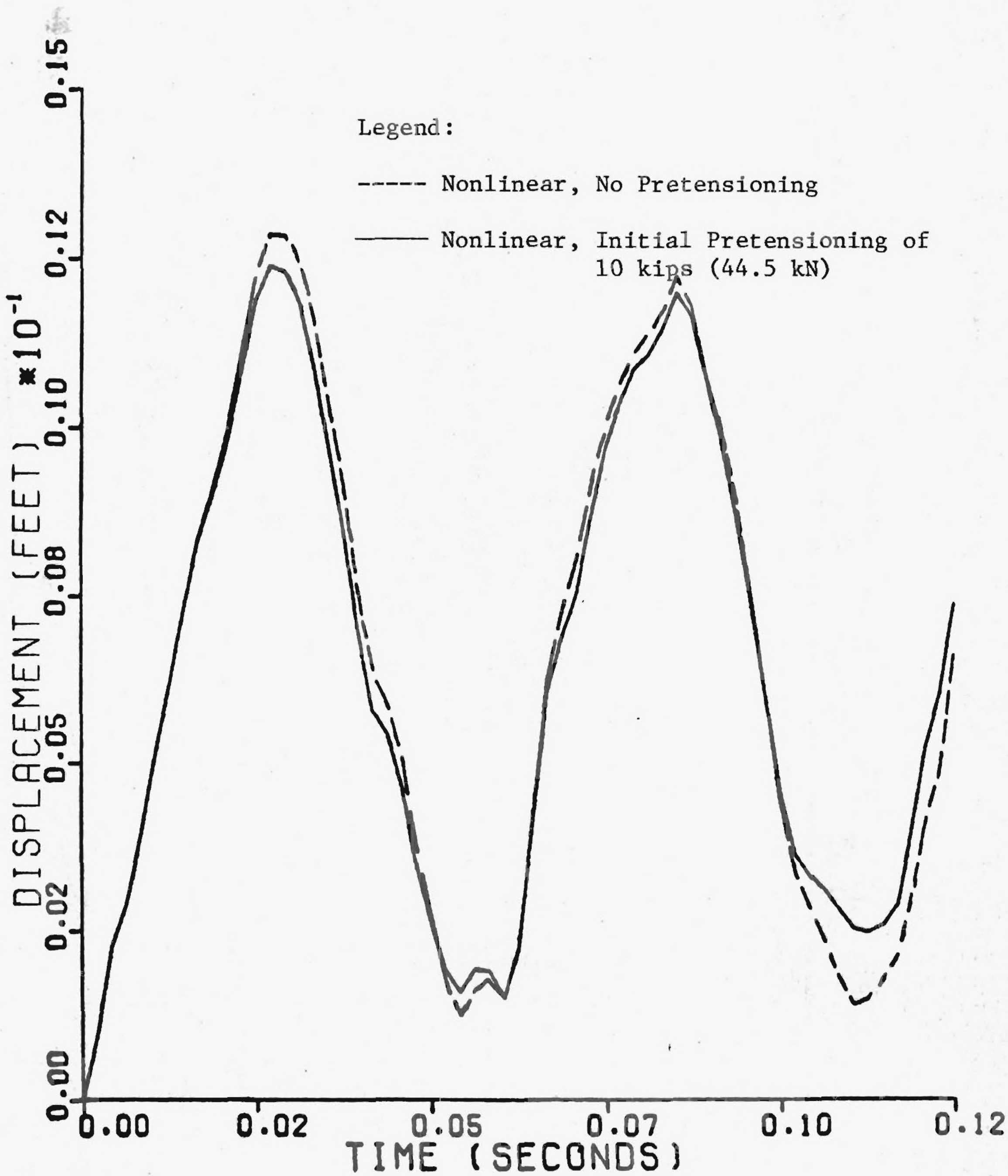



Figure 5.4-4 - Z-direction Undamped Dynamic Response at Joint 42 to Loading 1 With and Without Pretensioning of Tension-only Members.

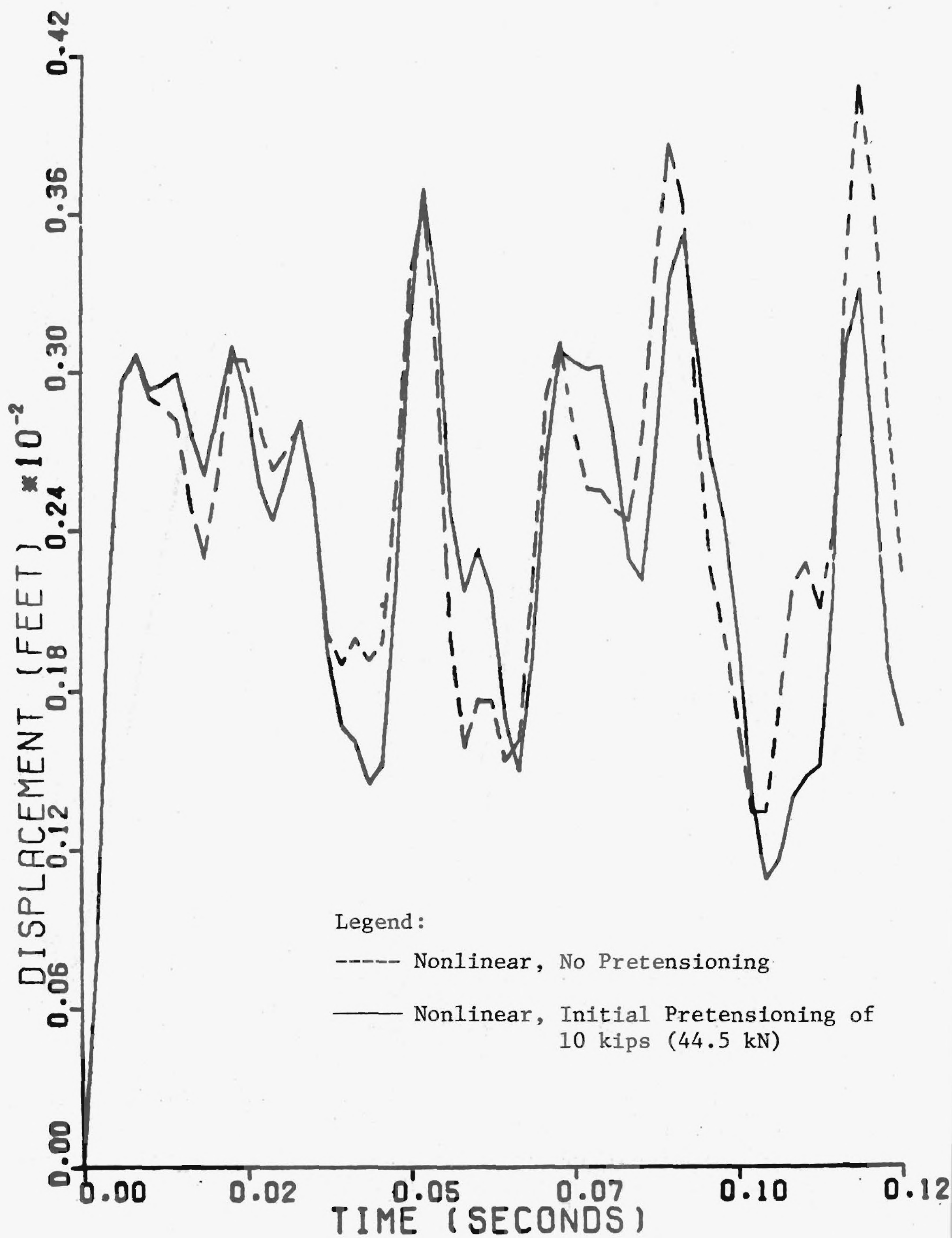



Figure 5.4-5 - X-direction Undamped Dynamic Response at Joint 42
Loading 2 With and Without Pretensioning of Tensic
only Members.

proportional damping matrix. First, the linear, damped and undamped response of the model containing actual cross-sectional areas for all tension-only members was investigated. Damped and undamped responses are compared in Figures 5.4-6 and 5.4-7 for each loading case. Then, damped and undamped responses were obtained using the nonlinear dynamic analysis approach and results for loadings 1 and 2 are contained in Figures 5.4-8 and 5.4-9. While both linear and nonlinear responses to loading 1 are only slightly affected, the linear and nonlinear response time-histories for loading 2 are greatly influenced by damping; much of the higher mode response was suppressed.

Finally, the linear and nonlinear damped responses are directly compared in Figures 5.4-10 and 5.4-11 for loadings 1 and 2, respectively. The responses are not significantly different for either loading case. Maximum displacement results for both loadings were still obtained using the nonlinear solution procedure, but in certain regions the linear solution is greater than the nonlinear solution for loading 2.

In general, it can be seen that even a small amount of damping had a greater influence on the dynamic response of the model than the nonlinear behavior or tension-only members, for the portion of the prototype structure and for the loadings considered.

## 5.5 Summary and Conclusions

An investigation was made into the use of linear and nonlinear solution procedures for the static and dynamic analysis of a section of the prototype structure containing tension-only members. Nonlinear solution procedures which accounted for the presence of tension-only members in an accurate manner were presented, and the degree of approximation involved in using linear solution procedures was evaluated by comparing linear and nonlinear results.

The nonlinear solution procedures which were employed to perform static and dynamic analyses were based on the iterative initial stress method. Direct linear extrapolation with the trapezoidal rule was used to integrate the equations of motion for dynamic analysis. Two approximate linear techniques were also used to analyze the tower structures. In the first, tension-only members were treated as standard tension-compression elements and were allowed to develop both tension and compression forces. Tension-only member cross-sectional areas were then reduced to negligible amounts

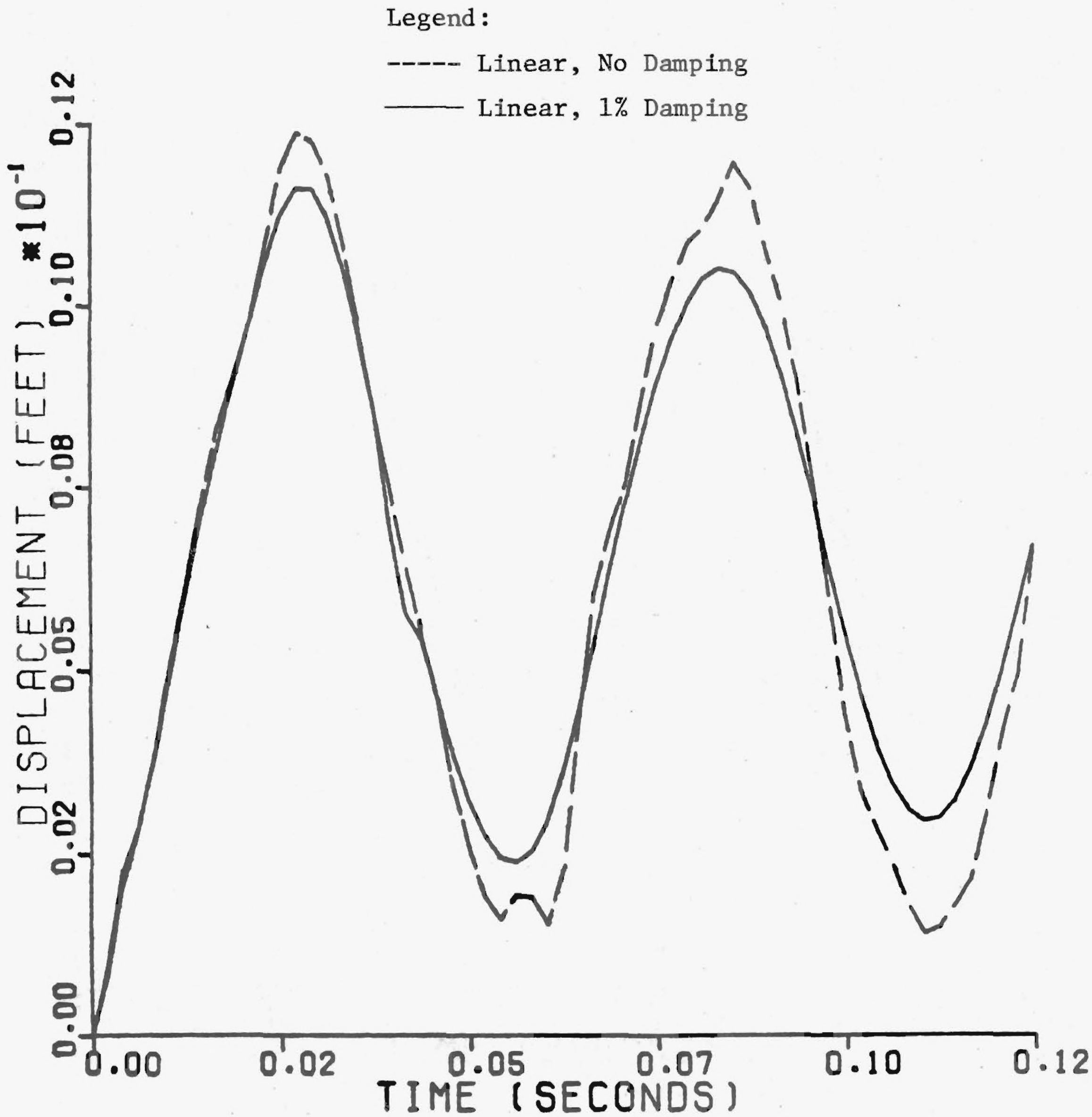



Figure 5.4-6 - Z-direction Dynamic Response at Joint 42 to Loading 1 With or Without Proportional Damping Using Linear Solution Procedures, No Pretensioning Applied.

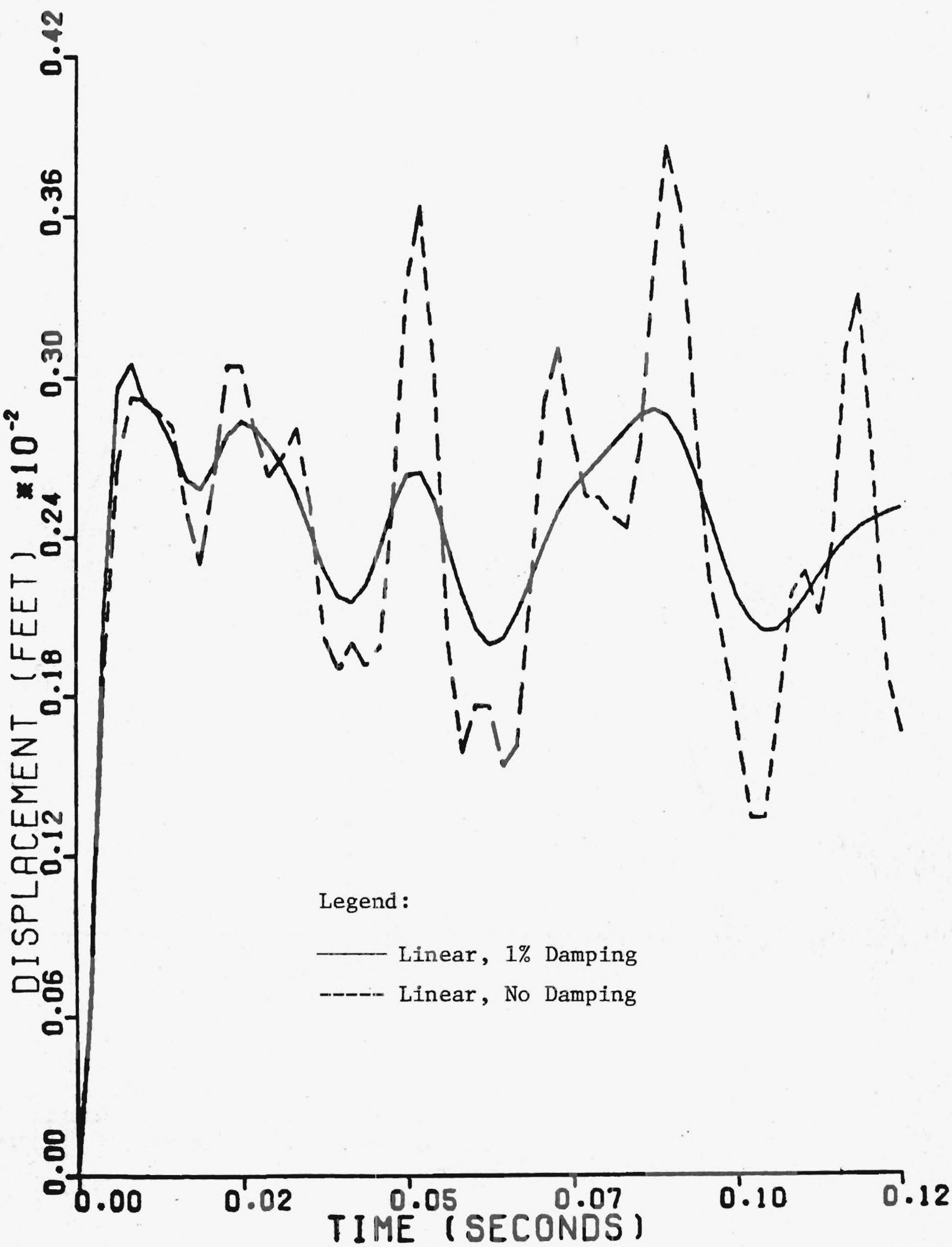



Figure 5.4-7 - X-direction Dynamic Response at Joint 42 to Loading 2 With or Without Proportional Damping Using Linear Solution Procedures, No Pretensioning Applied.

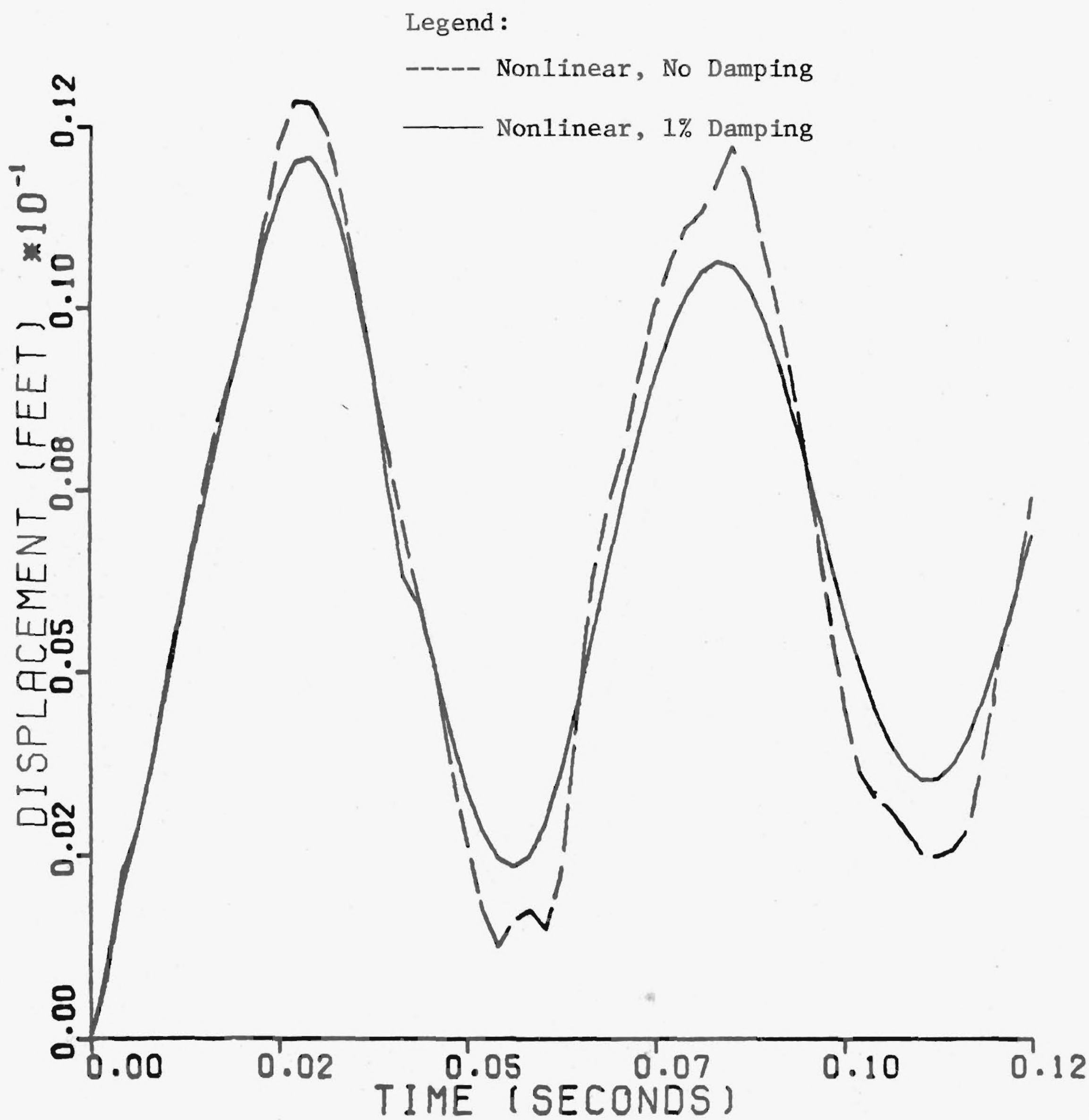



Figure 5.4-8 - Z-direction Dynamic Response at Joint 42 to Loading 1 With or Without Proportional Damping Using Nonlinear Solution Procedures, No Pretensioning Applied.

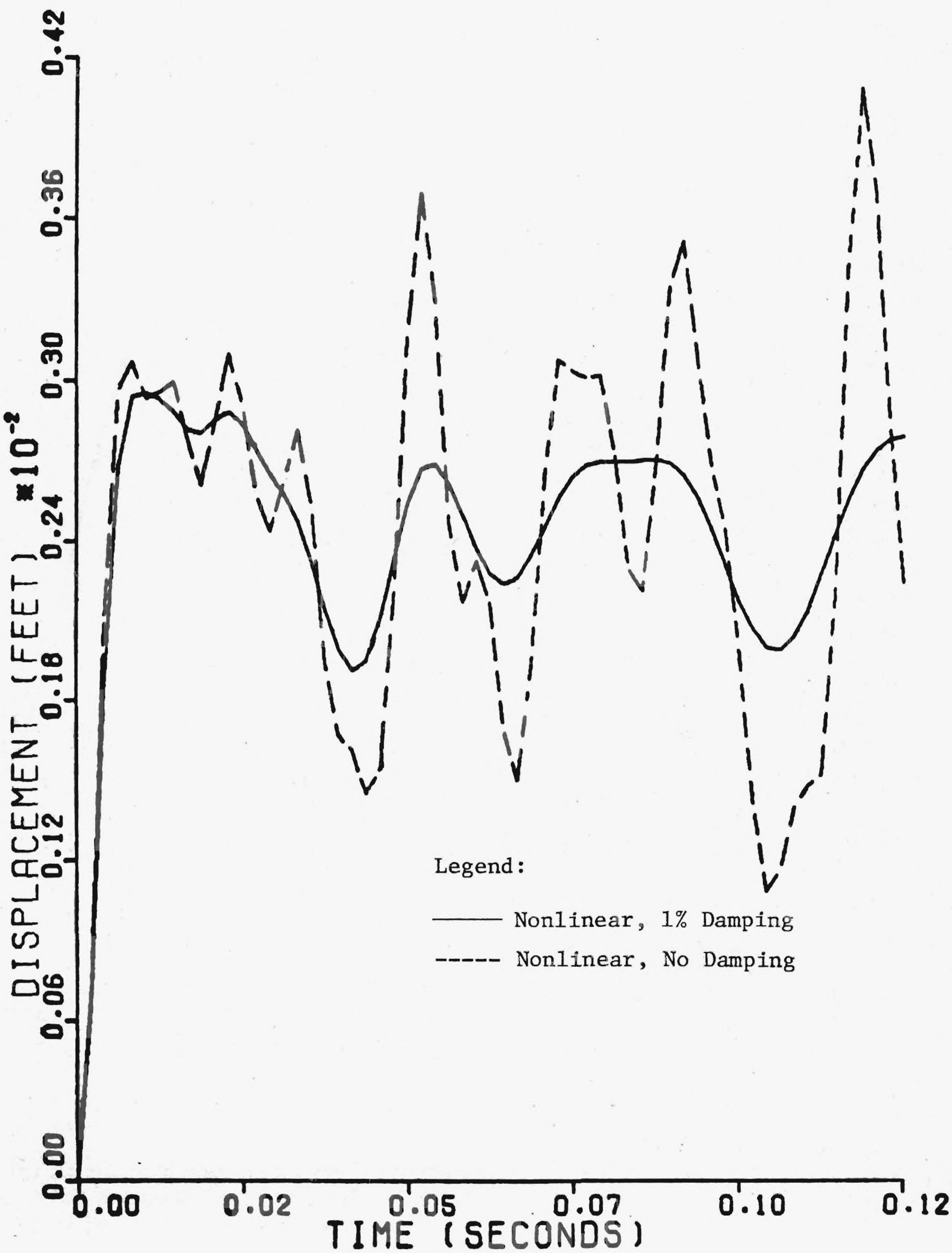



Figure 5.4-9 - X-direction Dynamic Response at Joint 42 to Loading 2 With or Without Proportional Damping Using Nonlinear Solution Procedures, No Pretensioning Applied.

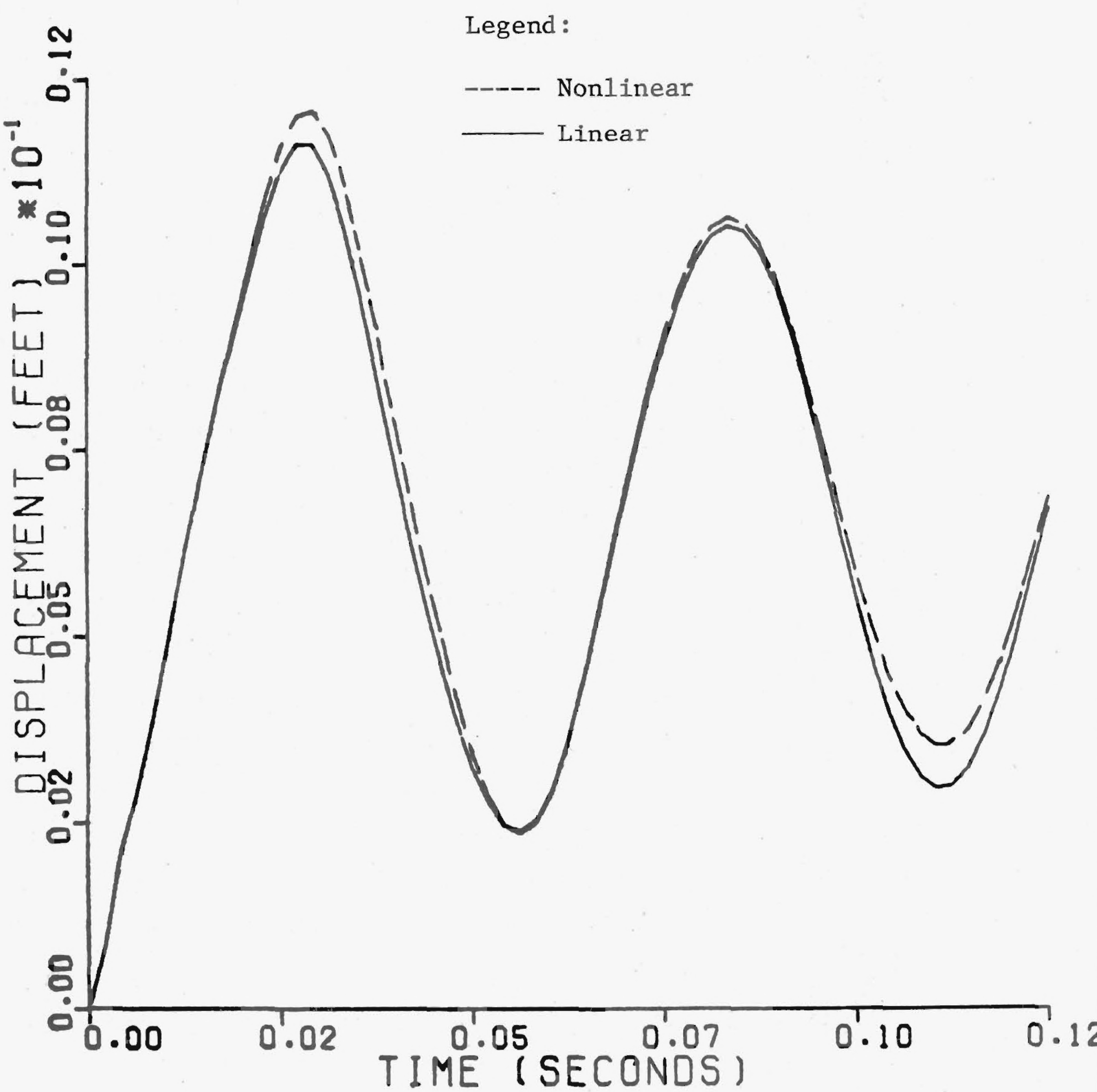



Figure 5.4-10 - Z-direction Dynamic Response at Joint 42 to Loadi
1 With 1% Proportional Damping From Linear and
Nonlinear Analyses, No Pretensioning Applied.

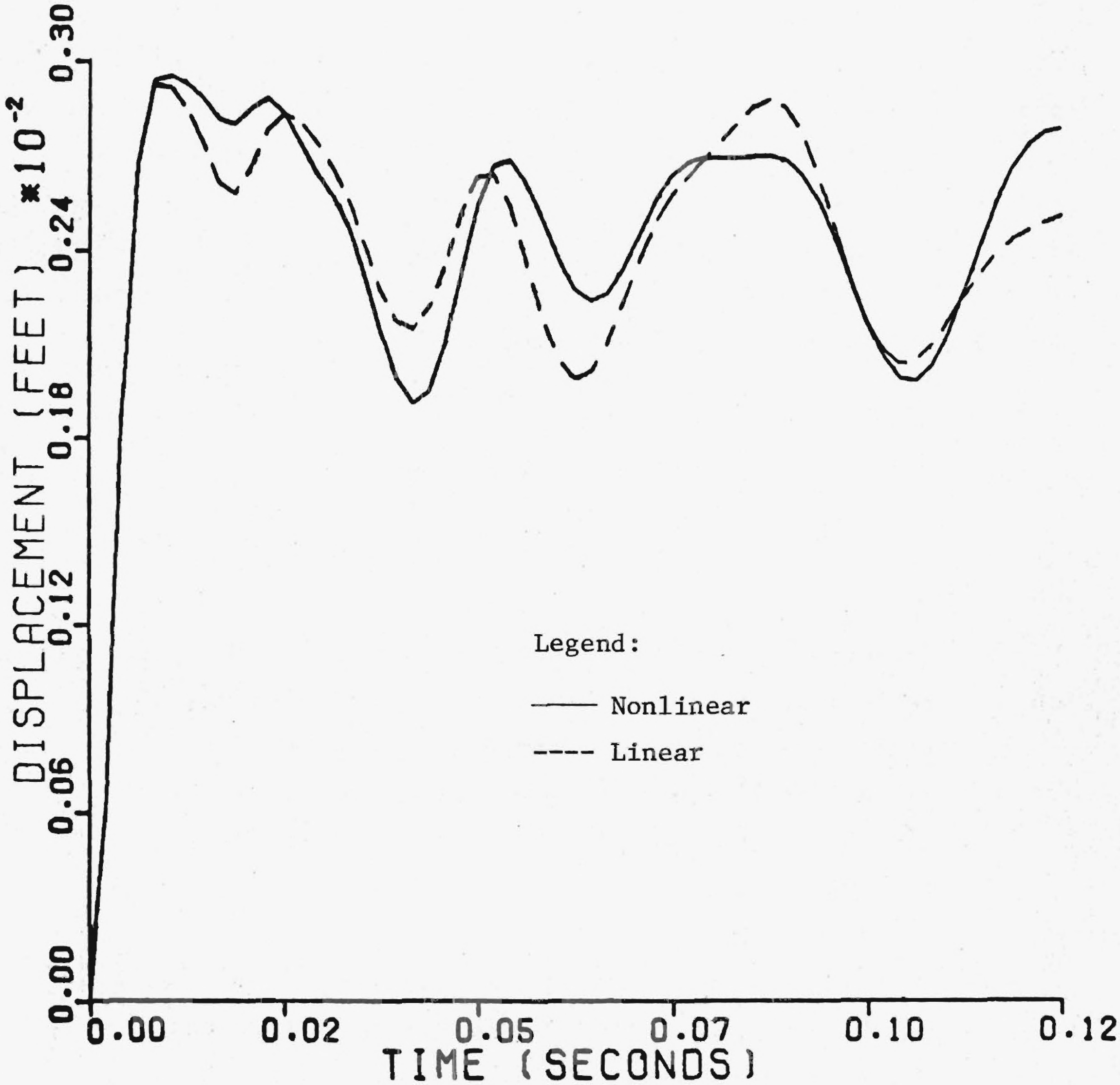



Figure 5.4-11 - X-direction Dynamic Response at Joint 42 to Loading 2 With 1% Proportional Damping From Linear and Nonlinear Analyses, No Pretensioning Applied.

for the second linear analysis, thereby removing their effect as force carrying members. Results of the linear and nonlinear solutions were compared for selected loading cases.

Parameter studies were conducted to determine the effect of pretensioning of tension-only members on overall structure response. The specified initial tension forces in tension-only members helped to balance compression forces developed by external loads, and various levels of pretensioning were studied for the tension-only members in the structures.

The relative effects of damping on dynamic response were also considered. Proportional damping matrices were developed for the structures, and linear and nonlinear analysis results were compared with the solutions obtained for undamped structure response.

In general, the following conclusions can be drawn:

1) The true nonlinear static response behavior of the model was bounded above and below by the two approximate linear models and linear solution procedures.
2) The effect of the nonlinear behavior of tension-only members on the static response of section AB was small; the maximum difference in linear and nonlinear analysis displacement results was less than 6% for the two loading conditions considered.
3) Pretensioning of tension-only members reduced overall structure displacement response.
4) For initial pretensioning forces in tension-only members which were greater than compression forces caused by external loads, nonlinear displacement response was equivalent to linear displacement response with original tension-only member cross-sectional areas specified.
5) The maximum dynamic response of the model, obtained from nonlinear solution procedures, was bounded by the peak displacements of the two approximate linear models, but selected segments of the response-time histories were not bounded for the two loadings considered.

6) A relatively small amount of damping influenced the dynamic response of the model more than the nonlinear behavior of tension-only members.

7) Damping had little effect in changing the difference between linear and nonlinear response results when compared to the undamped case.

## Chapter 6
## CONCLUSIONS

### 6.1 Summary

The dynamic properties and performance of large, self-supporting, latticed-steel communications towers acted upon by dynamic loadings were investigated in this study. Two towers, referred to as the large and small prototype structures, were studied in detail. A number of analytical models were developed for the towers using several different substructuring techniques. In general, the towers were modeled as space trusses pinned at the base in which the structural members were assumed to resist axial loads only. The presence of tension-only members was accounted for in the models in an approximate manner, and planar joints were properly supported to ensure a stable structure.

Two analysis approaches were used. In the first, the direct assemblage approach, the elastic properties of the prototype towers were developed using GTSTRUDL and were presented in the form of condensed stiffness matrices for preselected master degrees of freedom. Unit displacements were introduced at the master degrees of freedom and a condensed stiffness matrix assembled from the reaction forces. Tower mass tributary to the substructure boundaries was lumped at the master degrees of freedom to form the dynamic model. Parameter studies were conducted and natural frequencies and mode shapes determined for a variety of tower models.

The second approach involved a general substructuring procedure for assemblage of condensed dynamic models of free-standing tower structures. The procedure is based upon series elimination of unessential displacement coordinates, and employs the modified tridiagonal method used in analysis of tier buildings. The reduced dynamic model has mass and stiffness terms associated with preselected master degrees of freedom, only, and constitutes an efficient system for dynamic response computations.

Using the dynamic models, the effect of add-on dampers on the seismic response of the large prototype structure was investigated. Direct linear extrapolation with the trapezoidal rule was used to integrate the equations of motion. Only one seismic loading was considered to simplify the analysis and to permit a wider range of damping cases to be compared. The number, size and distribution of dampers required to produce a significant reduction in structure response were studied.

An investigation was made into the use of linear and nonlinear solution procedures for the static and dynamic analysis of free-standing towers with tension-only members. Nonlinear solution procedures which accounted for the true nonlinear behavior of tension-only members were presented, and the degree of approximation involved in using linear solution procedures was evaluated by comparing linear and nonlinear results.

Static analysis results were obtained using an iterative initial-stress solution procedure. Dynamic response results were obtained using direct linear extrapolation in a step-iterative procedure. In addition, two approximate linear techniques were also used to analyze the tower structures. In the first, tension-only members were treated as standard tension-compression elements and were allowed to develop both tension and compression forces. Tension-only member cross-sectional areas were then reduced to negligible amounts for the second linear analysis, thereby removing their effect as force carrying members. Results of the linear and nonlinear solutions were compared for selected loading cases. A typical section of the large tower was selected for analysis to observe the effect of tension-only members on the response of an actual structure. Several torsional and translational loading conditions were applied to the section, and static and dynamic response was determined using the linear and nonlinear solution procedures.

Parameter studies were conducted to determine the effect of pretensioning of tension-only members on overall structure response. The specified initial tension forces in tension-only members helped to balance compression forces developed by external loads, and various levels of pretensioning were studied for the tension-only members in the structure.

The relative effects of damping on dynamic response were also considered. Proportional damping matrices were developed for the structure, and linear and nonlinear analysis results were compared with the solutions obtained for undamped structure response.

## 6.2 Conclusions and Recommendations

Analytical Models. - - Substructuring for analysis of large and complex structures, such as free-standing towers, was once again shown to be a flexible and economical means of dealing with the large number of degrees of freedom present in such systems. The combination of a series elimination procedure with the capability for arbitrary selection of master degrees of freedom provides a new approach to dynamic response analysis of large towers.

Analytical frequencies and mode shapes for the actual large prototype structure were presented in Chapter 3. These values should be verified by conducting a program of full-scale measurements. Accelerometers attached to the tower could be used to detect ambient level motions of the tower and the data recorded on magnetic tape for later time series analysis. Studies of this kind have been attempted but have not been successful to date due to heavy signal interference from transmission equipment on the large tower.

In future analytical studies, it remains to extend the substructure procedure to handle dynamic analysis of arbitrarily-supported structures modeled by finite elements or finite element substructures (superelements).

The proper selection of the master degrees of freedom is expected to be more critical for arbitrary structures than for slender towers; and several different reduced models may have to be assembled to investigate the sensitivity of the structure to the location of master degrees of freedom.

Damping Studies. - - The influence of add-on dampers on the seismic response of the large tower was presented in Chapter 4. A viscous dashpot model was used to represent a typical damping device such as a shock absorber connected to diagonal bracing elements. In general, the degree of attenuation in the dynamic response of the large tower depended on the size and distribution of dampers and, to a lesser extent, on the number of such add-on devices.

Future studies of the effects of add-on dampers should consider their frequency and amplitude-dependent characteristics, useful life, and effectiveness for actual loadings. This may involve some experimental studies of several commercially-available shock-absorbing devices, and consideration of other damper models presented in the literature.

Finally, study of tower response to seismic loading showed that the displacement and velocity response were much greater in the upper more flexible part of the tower than in the region near the base. Therefore, it was noted that add-on dampers were likely to be more effective if they were placed near the top of the structure. However, the dynamic response study was performed using one component of one earthquake ground motion only. Additional earthquake records and other forcing functions such as wind which could excite other modes in the prototype structure need to be considered in further studies.

Nonlinear Studies. - - The true nonlinear behavior of tension-only members was considered in static and dynamic response analyses of a portion of the large tower. The influence of tension-only members was shown to be less important than the effect of internal damping for the model, loadings, and damping levels assumed. This conclusion may not hold for other structures with tension-only members with varying amounts of applied pretension force. Future studies concerned with the nonlinear behavior of slender towers should consider other sources of nonlinearity, such as material and geometric effects.

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## Appendix A

Three Dimensional Views of
Sections of Large Tower

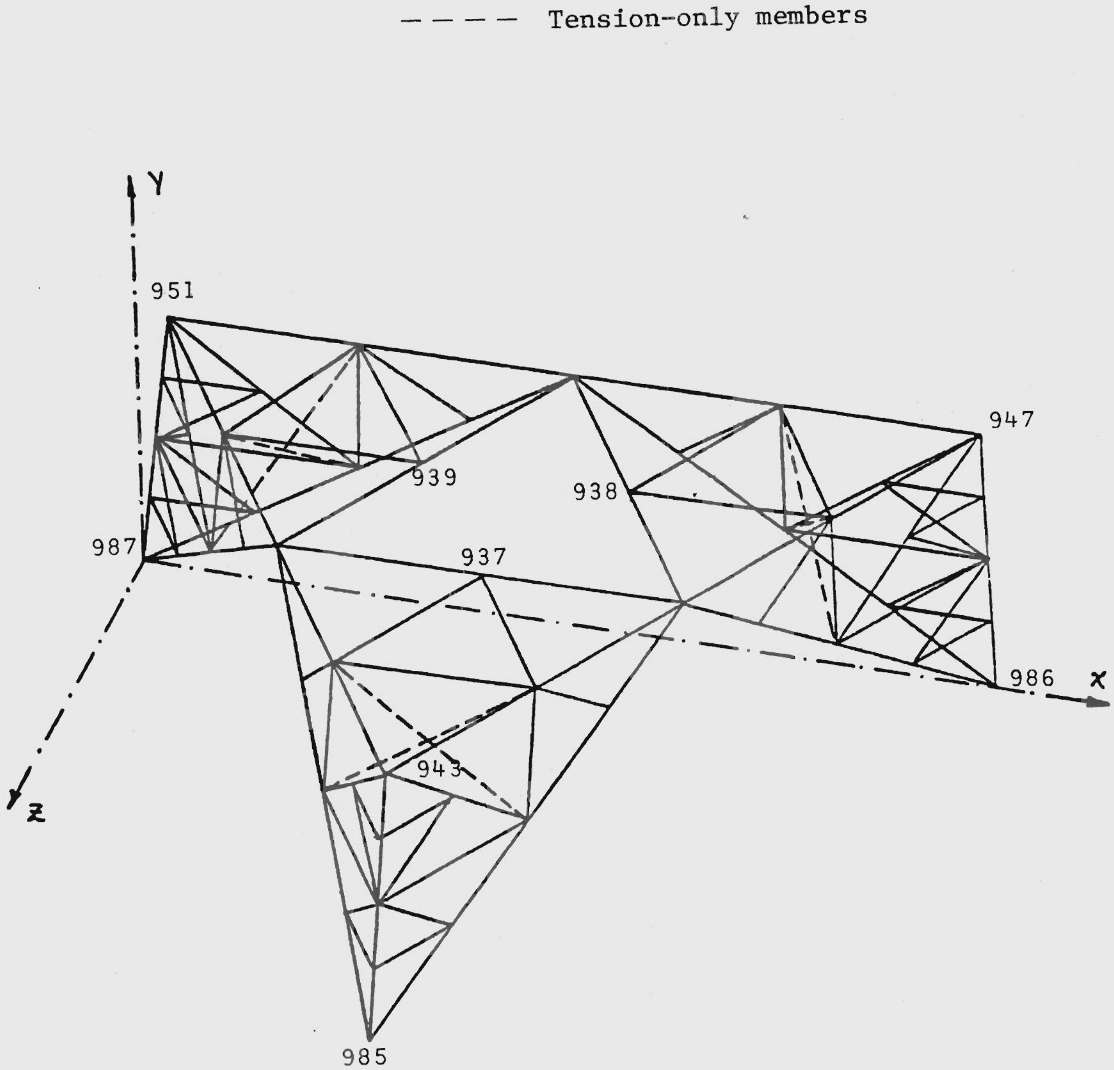


Figure A.1 - Section AI

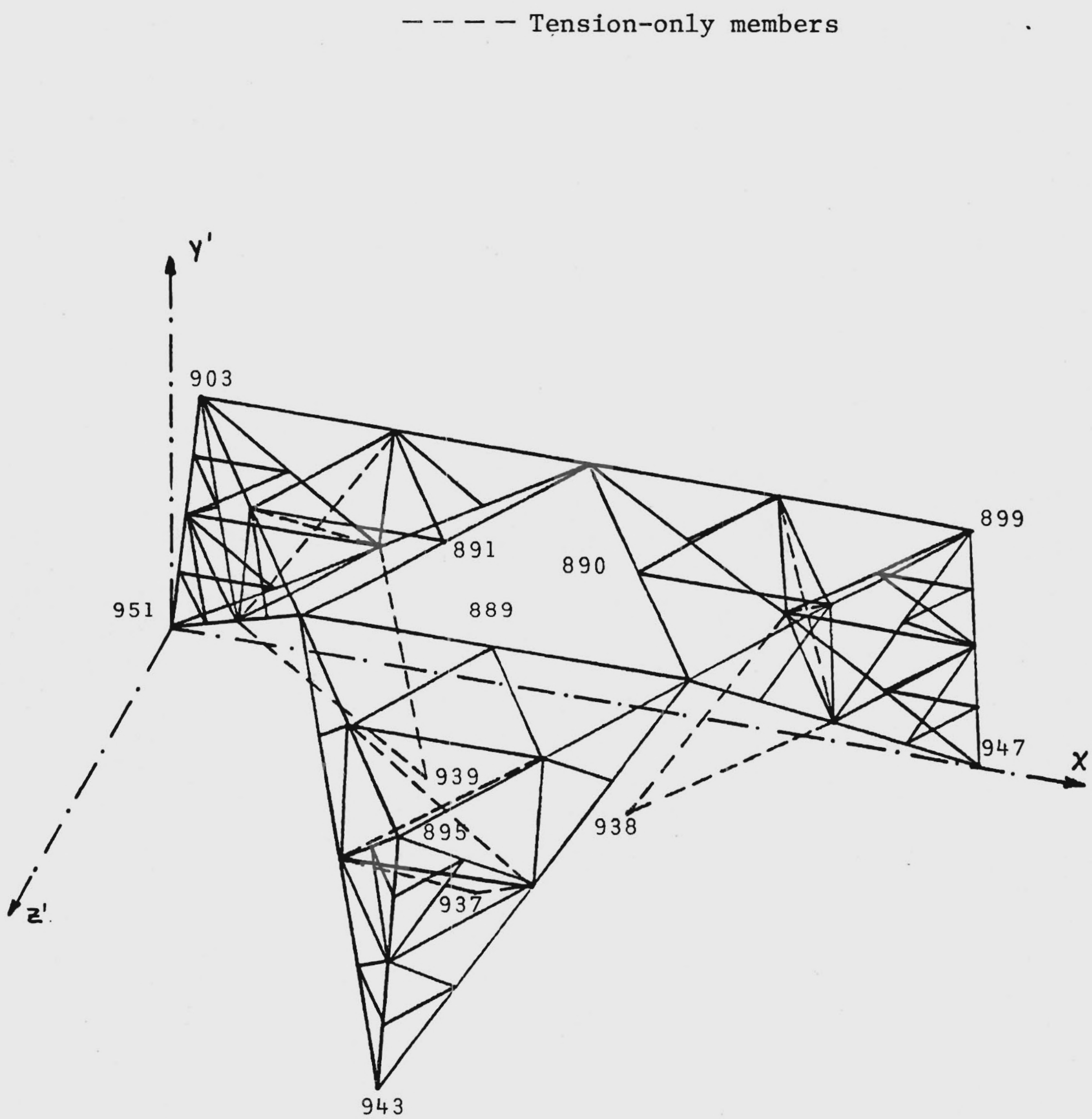


Figure A.2 - Section AH, Typical View
for Sections AA to AH

— — — — Tension-only members

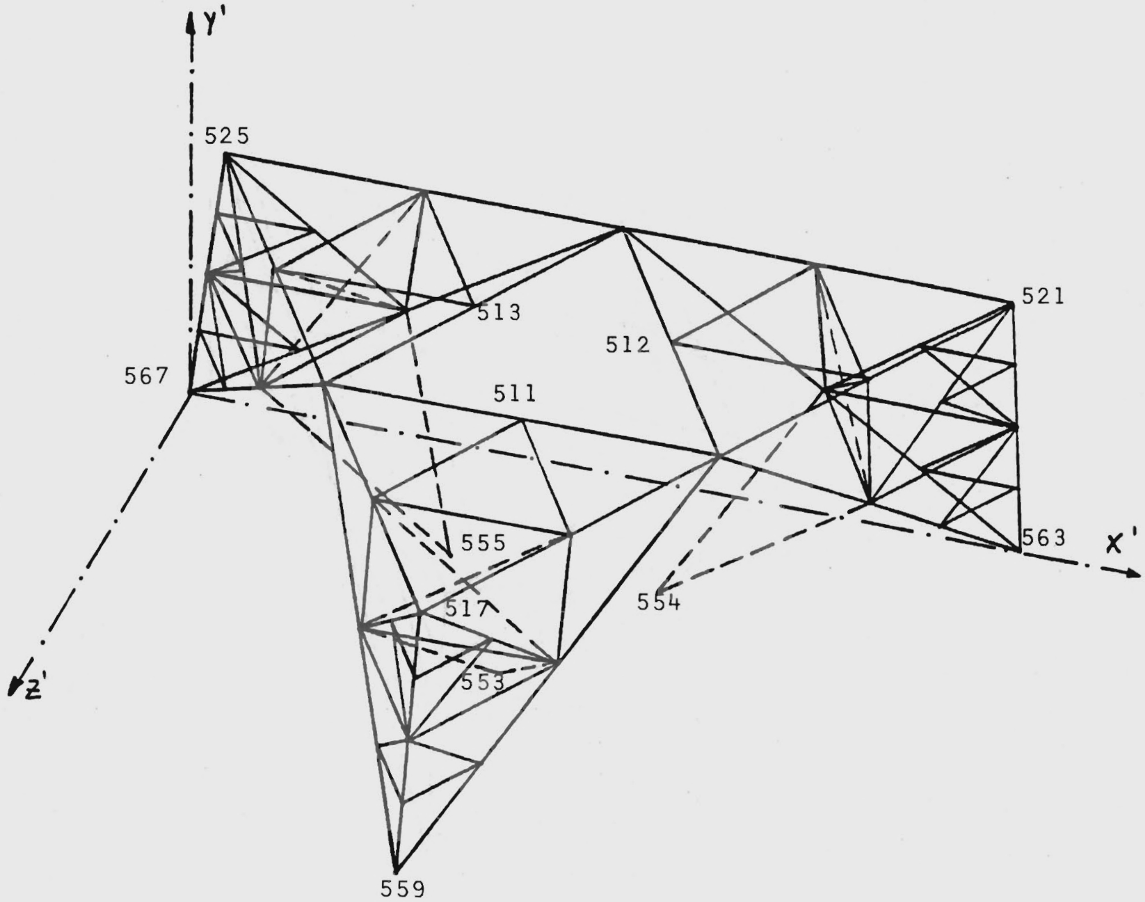


Figure A.3 - Section ZB

---- Tension-only members

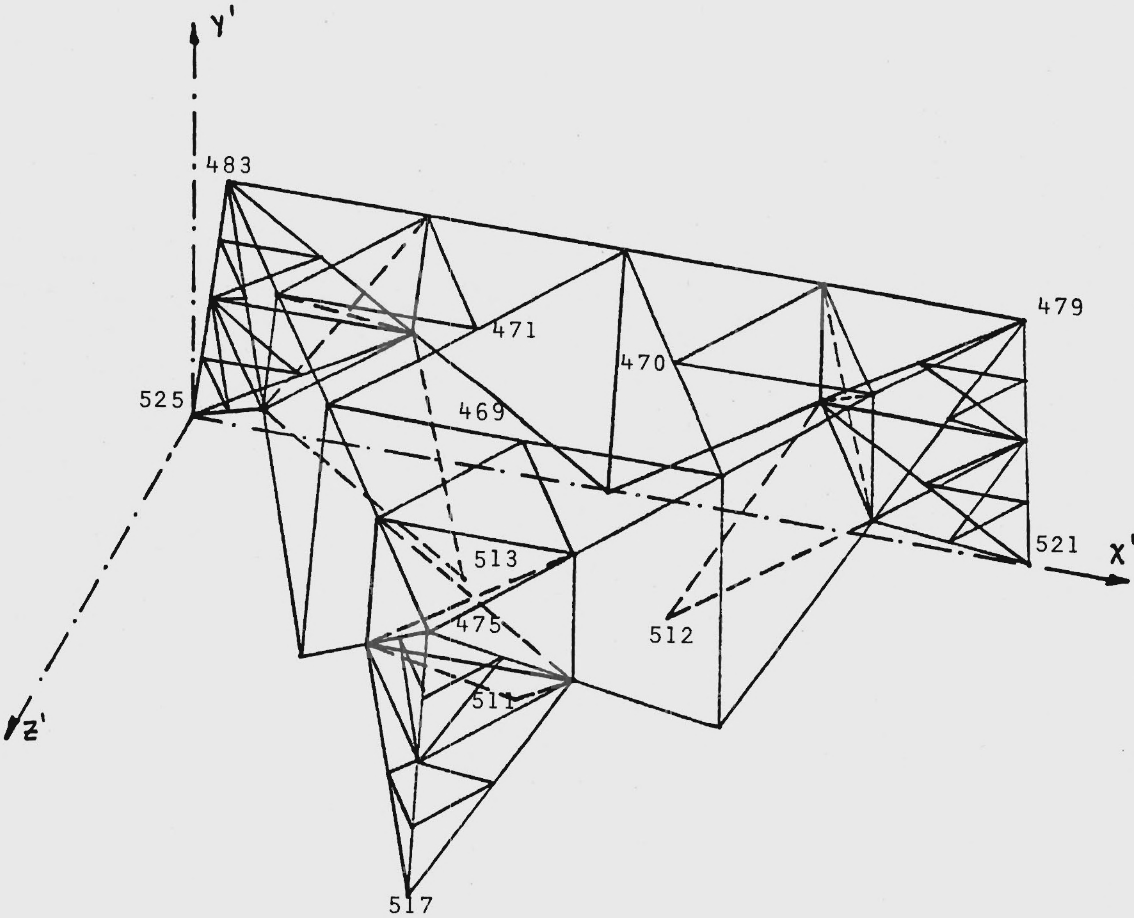


Figure A.4 - Section ZA

– – – – Tension-only members

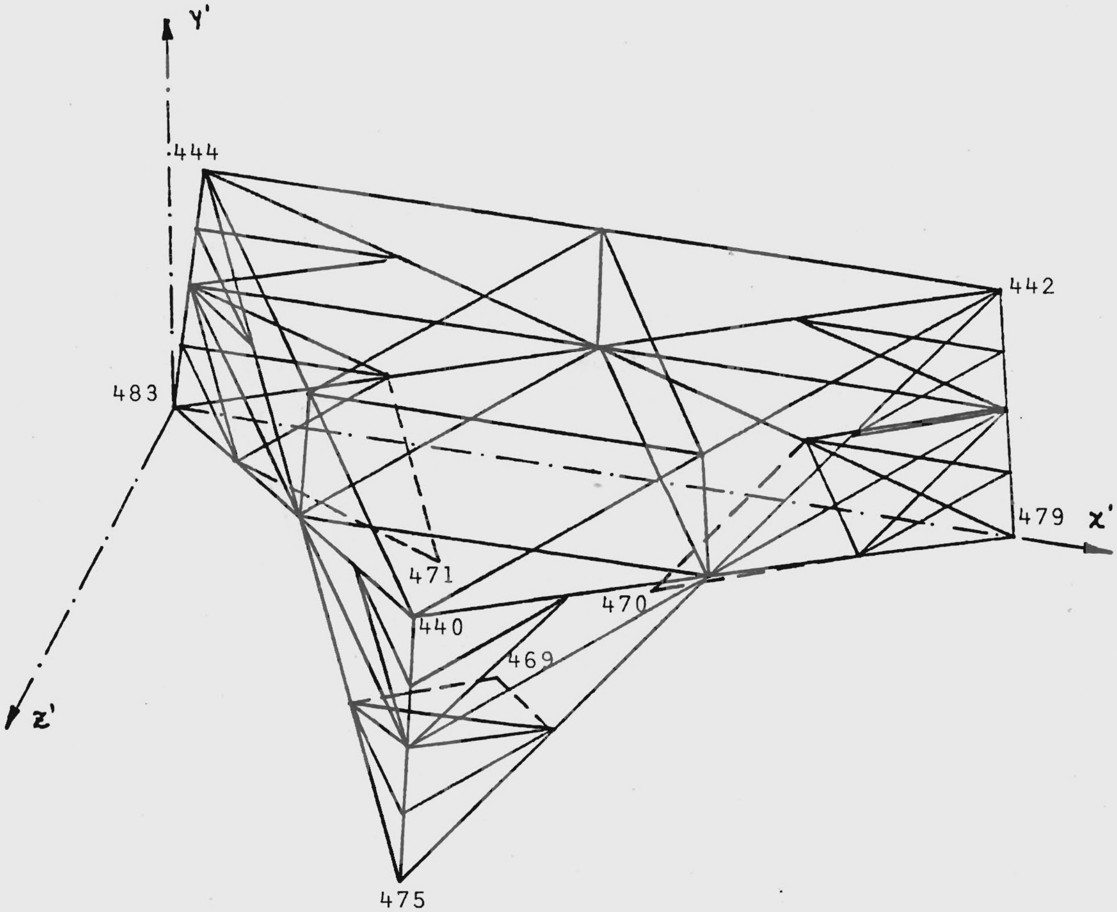


Figure A.5 - Section Y

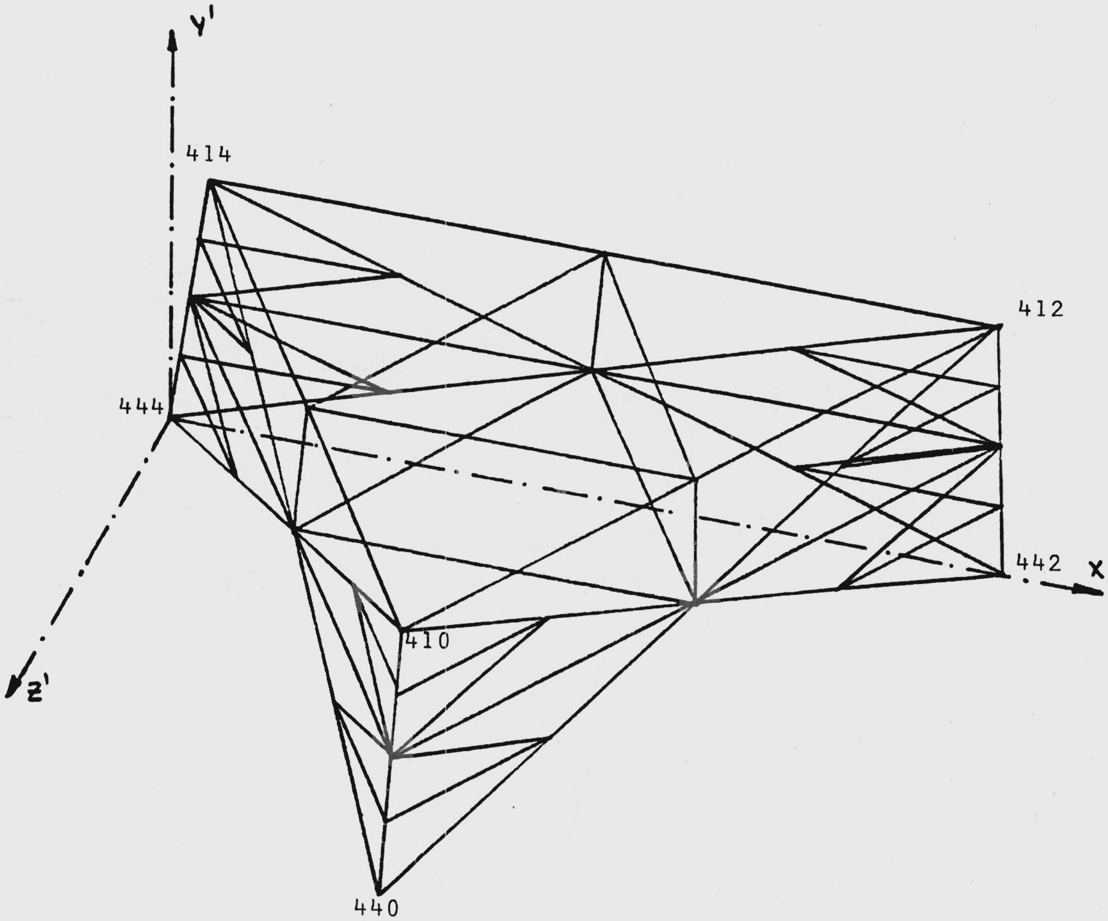


Figure A.6 - Section X

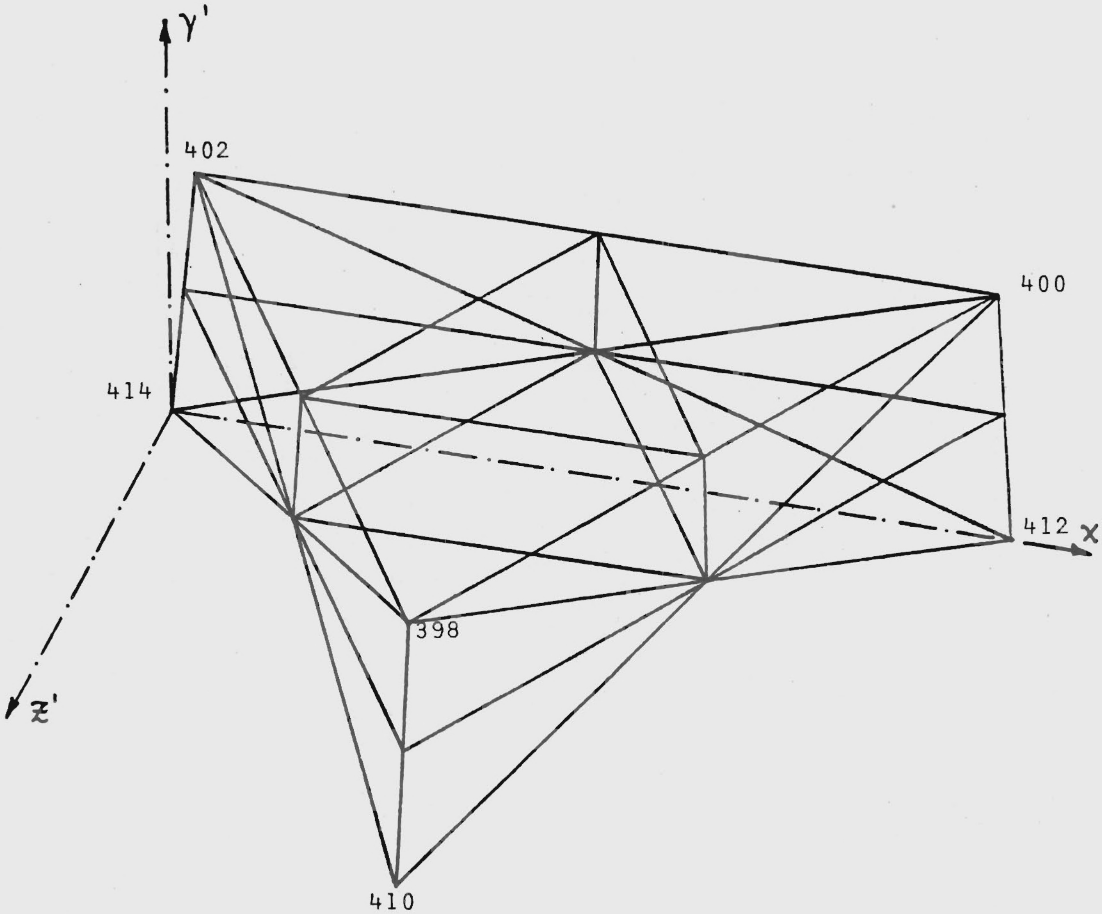


Figure A.7 - Section W, Typical View
for Sections V,W,S,T,Q,R

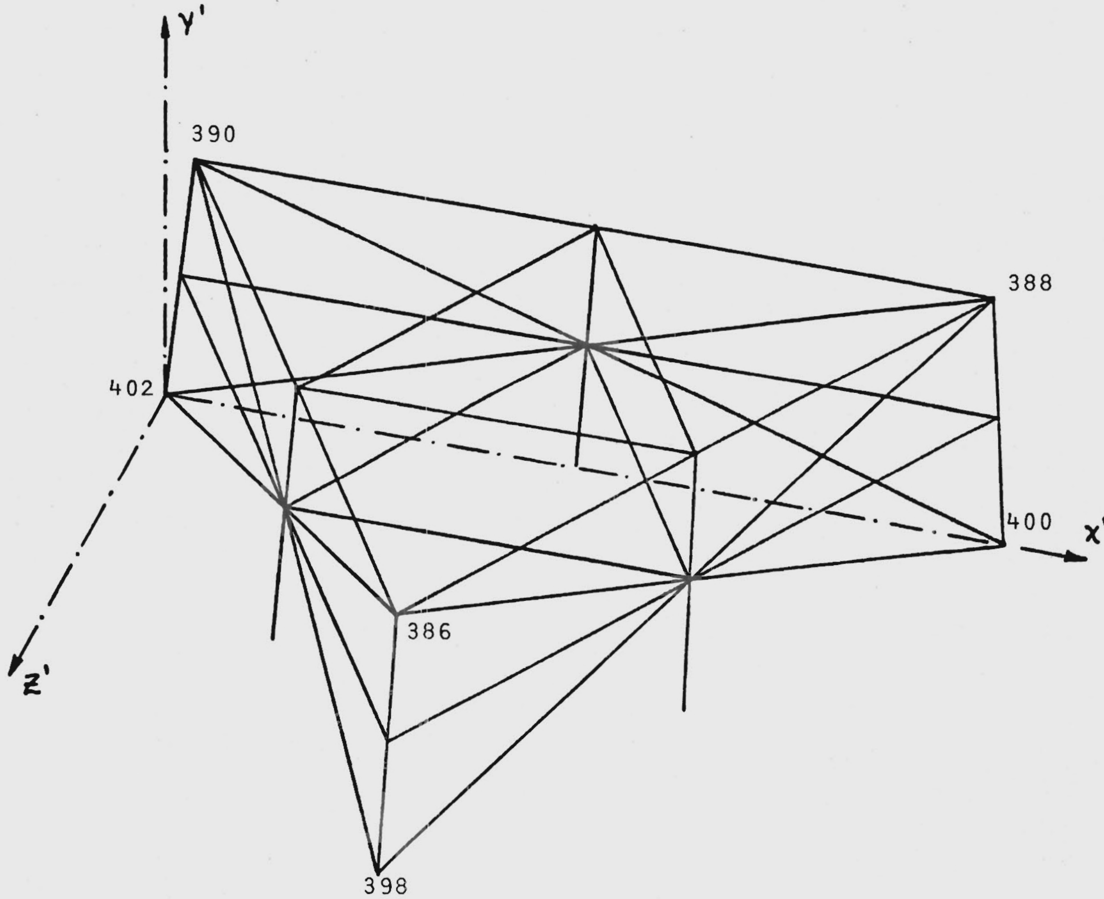


Figure A.8 - Section V*, Typical View
for Sections V*,S*,Q*

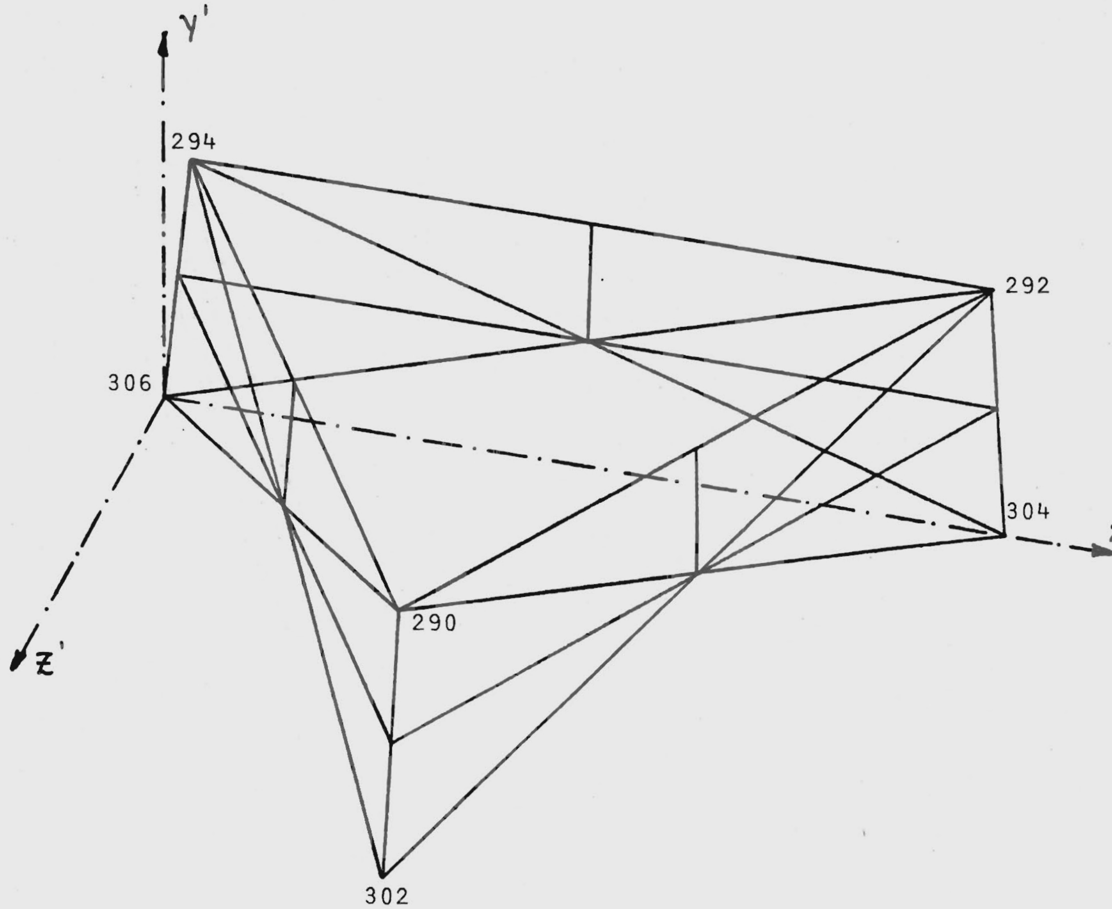


Figure A.9 - Section P, Typical View
for Sections P,M,L,J

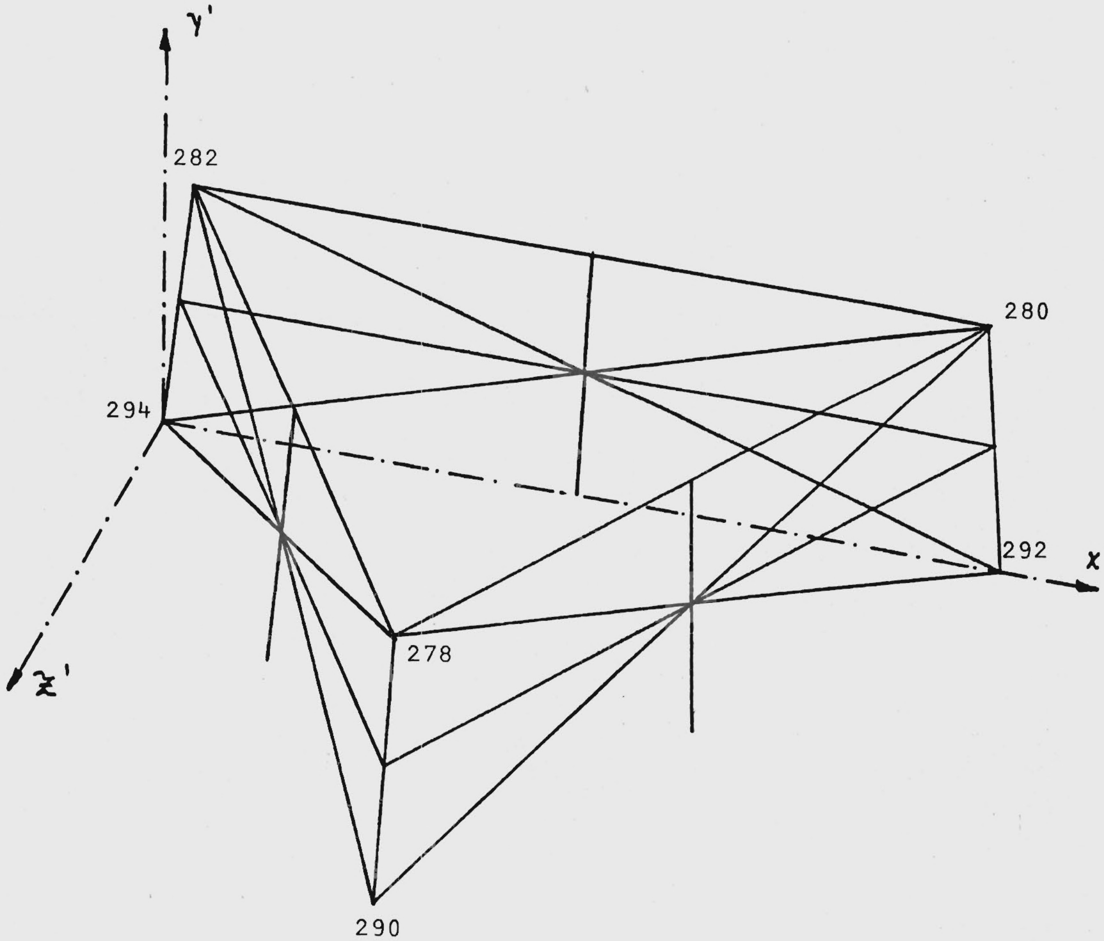


Figure A.10 - Section N, Typical View
for Sections N,K

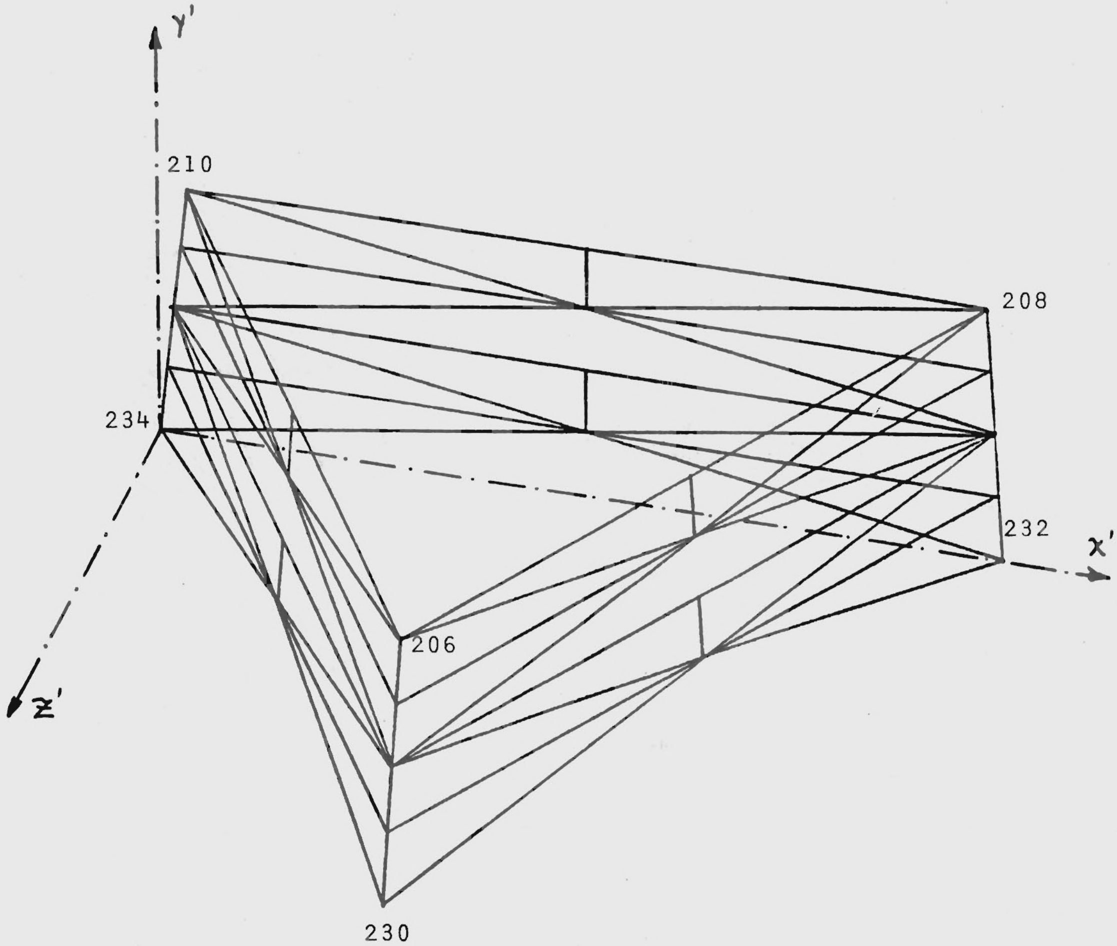


Figure A.11 - Section H, Typical View
for Sections F,G,H

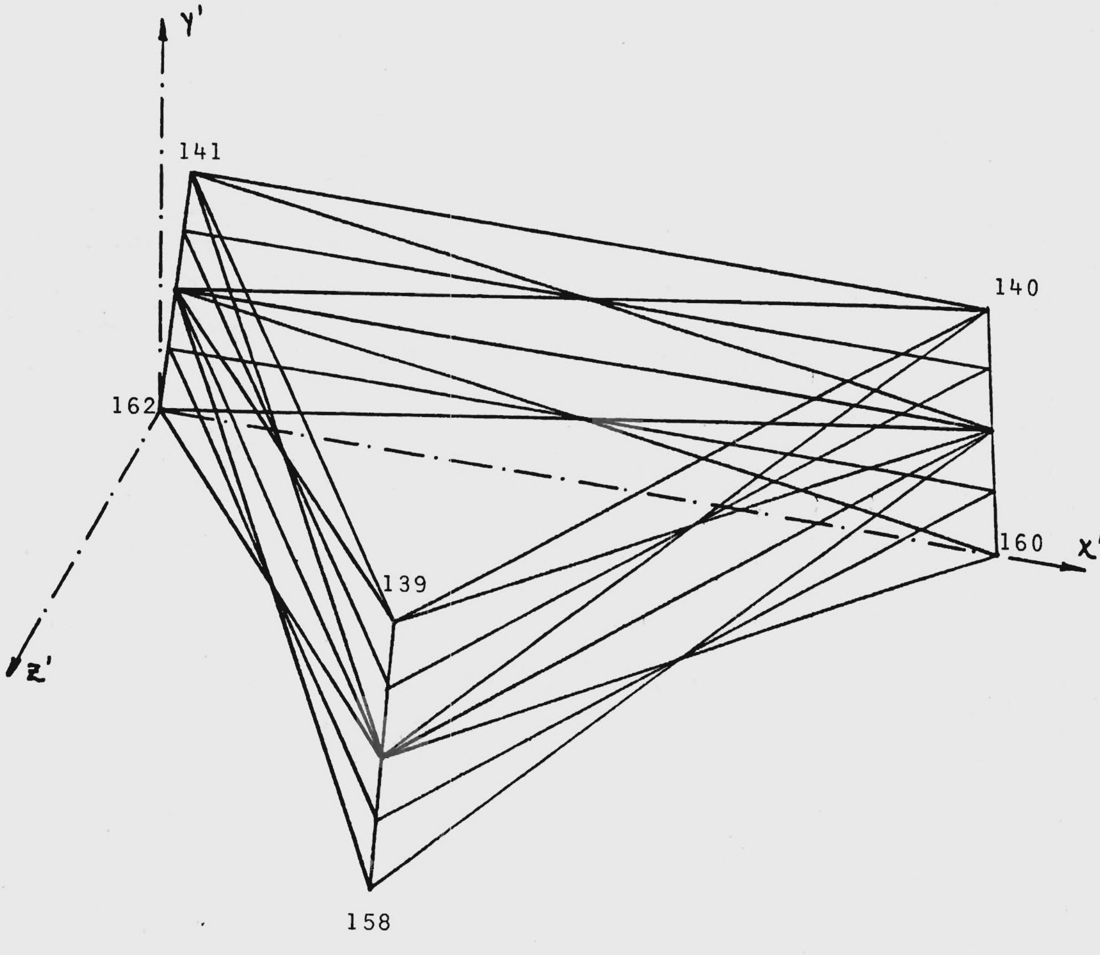


Figure A.12 - Section E

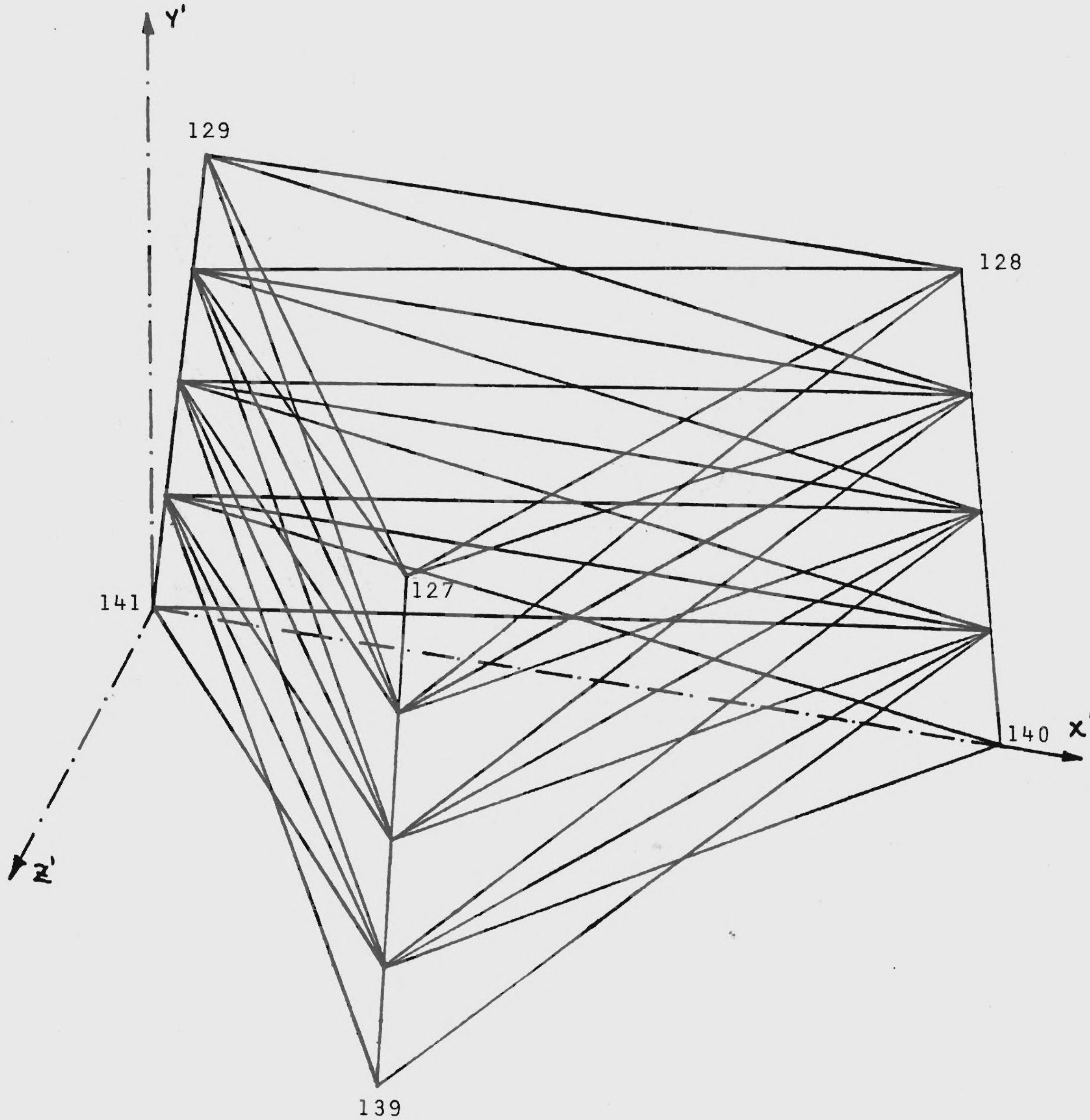


Figure A.13 - Section D, Typical View for Sections D,C

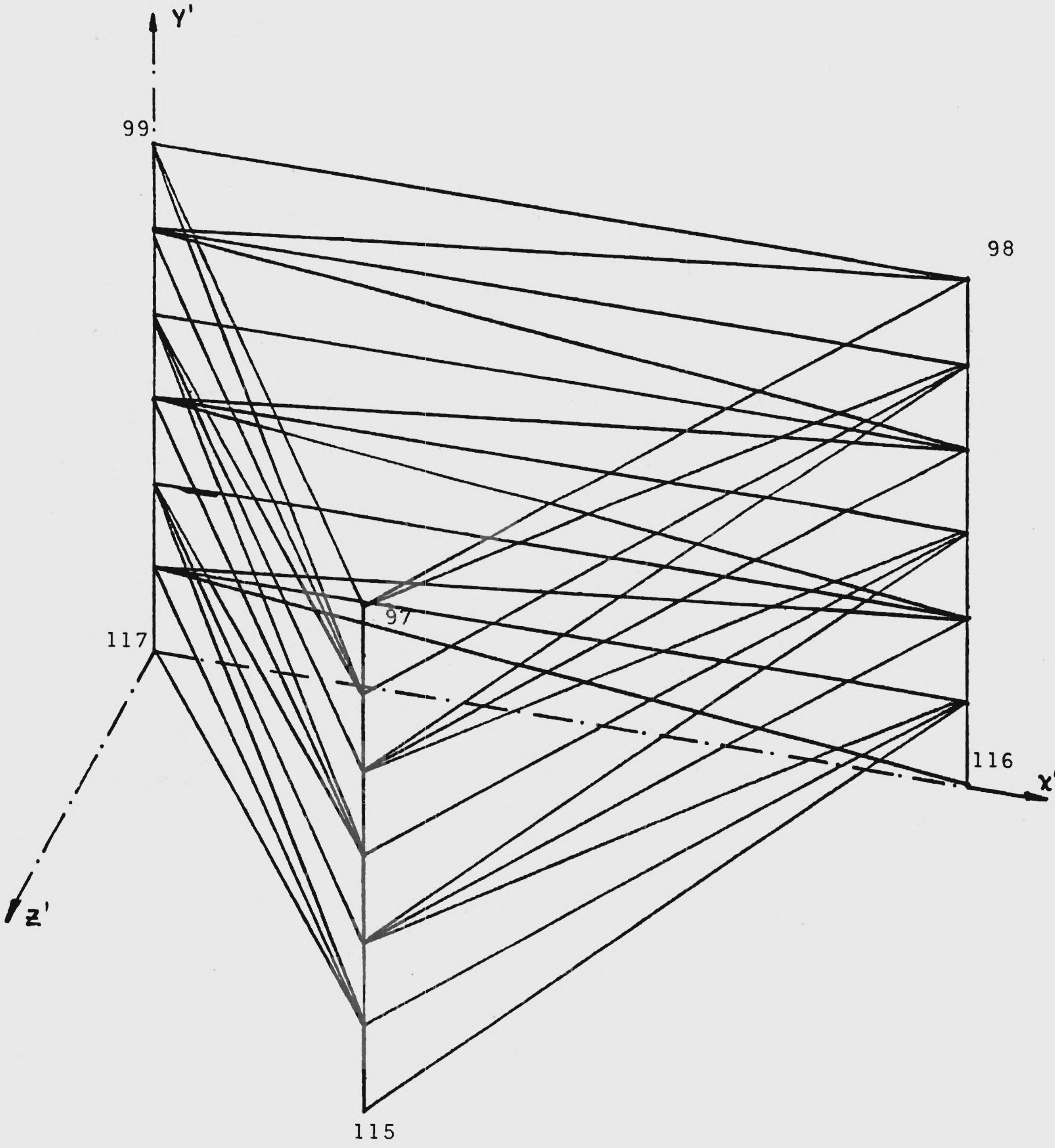


Figure A.14 - Section 6, Typical View for Sections 1 to 4

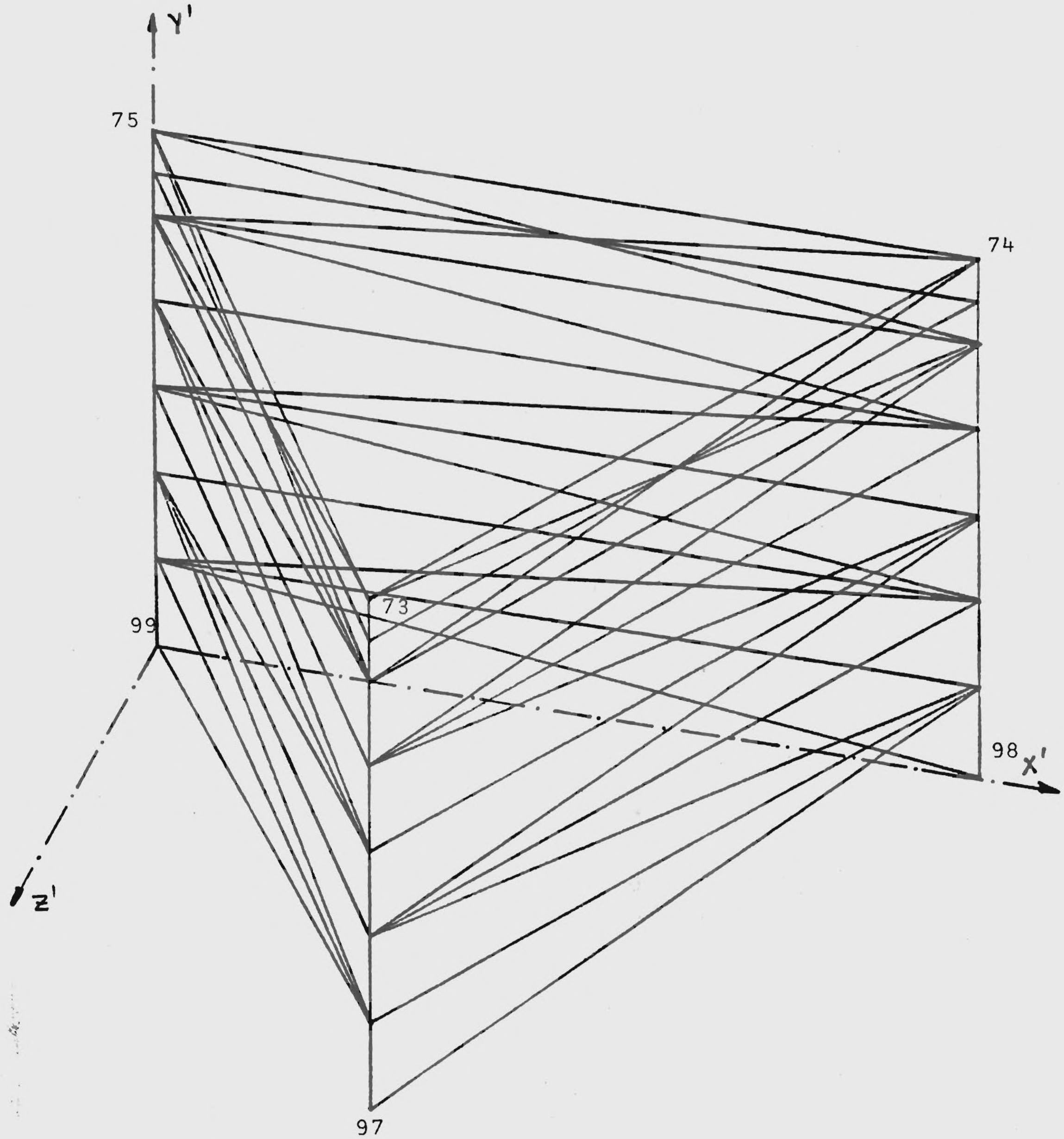


Figure A.15 - Section 5

# Appendix B

Program TOWER

Table B.1 - Data Input Guide for Program TOWER

| Data Item* | Number of Cards | Description |
|---|---|---|
| 1. NSUBS | 1 | Number of substructures |
| | (Repeat items 2 to 5 "NSUBS" times) | |
| 2. SSN,JTNOMX, NSUM,JTBI, NJWFT,E,UW | 1 | Substructure no.,maximum joint no., no. of members, joint no. of first B joint, no. of joints with F types, elastic modulus, unit weight |
| 3. K,X( ),Y( ), Z( ) | NSUFTS (no. of substructure joints) | Joint no., coordinates |
| 4. J,NFL(x), NFL(y),NFL(z) | NJWFT | Joint no., master DOF list for x,y,z directio (1-F type, 2-B to F type, 0-A, or B to A type |
| 5. J,JJ( ),JK( ), AX( ) | NSUM | Member no., start and end joint nos., cross-sectional area |
| 6. MOPT | 1 | Mass option code (1-CM, 2-ALM) |
| | (Repeat items 7,8 "NSUBS" times) | |
| 7. NJWALM | 1 | No. of joints with added lumped mass |
| 8. JAM( ),X( ) | NJWALM | Joint no., added mass |

*Free format input.

```
*DECK TOWER
      PROGRAM TOWER(INPUT,OUTPUT,TEMPS,TDATA,TOWRSM,TAPE5=INPUT,
     #              TAPE10=TEMPS,TAPE11=TDATA,TAPE12=TOWRSM)
C
C
C-----SUBSTRUCTURE ANALYSIS OF FREE-STANDING TOWERS
C
C-----PROGRAMMER:  DR. B. J. GOODNO             1977
C                  SCHOOL OF CIVIL ENGINEERING
C                  GEORGIA INSTITUTE OF TECHNOLOGY
C                  ATLANTA, GEORGIA  30332
C                  (404)894-2227
C
C-----DESCRIPTION:
C         PROGRAM TOWER ASSEMBLES STRUCTURE STIFFNESS AND MASS
C         MATRICES FOR A FREE-STANDING TOWER STRUCTURE MODELED AS A
C         LINEARLY-ELASTIC SPACE TRUSS.  THE TOWER IS ASSUMED TO BE
C         AN ASSEMBLAGE OF SUBSTRUCTURES WITH BOUNDARIES BETWEEN
C         SUBSTRUCTURES SELECTED BY THE ANALYST.  A FORWARD ELIMINATION
C         SEQUENCE IS USED TO REMOVE UNNECESSARY DEGREES OF FREEDOM
C         FROM THE DYNAMIC MODEL BUT MASTER DEGREES OF FREEDOM MAY BE
C         SPECIFIED ARBITRARILY AT ANY JOINT IN THE STRUCTURE.  EITHER
C         CONSISTENT OR LUMPED MASS MAY BE USED AND ADDITIONAL LUMPED
C         MASSES MAY BE LOCATED AT ANY JOINT IN THE SUBSTRUCTURE.  THE
C         STRUCTURE STIFFNESS AND MASS MATRICES ARE FORMED IN CONSISTENT
C         FASHION USING MATRIX CONDENSATION PROCEDURES.  FINALLY,
C         VIBRATION FREQUENCIES AND MODE SHAPES ARE COMPUTED FOR THE
C         REDUCED MODEL.
C
C
      REAL L
      INTEGER EPV,SUPV,OLDNM,OLDNFT,OLDB1,OLDNJAF,OLDNJBT
C
      COMMON
     1/CBPARMS/ISM,NSUBS,NSUN,NSUDF,NSUFT,NAT,NBT,NABT,KFF,
     #          OLDNFT,OLDNM,NM,JTB1,OLDB1,NDA1,NDB1,NDF1,
     #          NJWAFT,NJWBT,NSUJTS,OLDNJAF,OLDNJBT,MOPT,NJWALM,E,UW
     2/CBNM/L(130),AX(130),CX(130),CY(130),CZ(130),JJ(130),JK(130),
     #     SMD(6,6),SWT
     3/CBNDOF/NFL(171),NBL(171),SUPV(171),EPV(6),
     #         SAA(126,126),SAB(126,45),SAF(126,42),
     #                      SBB(45,45),SBF(45,42),
     #                                 SFF(42,42),
     #         TS1(126,45),TS2(126,42),TS3(126,126),TS4(126,42)
     4/CBNJ/JAM(57),X(57),Y(57),Z(57)
C
C-----ARRAY DIMENSION CONTROL PARAMETERS
C
C --- ARRAY SIZES: ARRAYS IN COMMON BLOCK CBNDOF
C           SAA:  NDA1 X NDA1
C           SAB:  NDA1 X NDB1
C           SAF:  NDA1 X NDF1
C           SBB:  NDB1 X NDB1
C           SBF:  NDB1 X NDF1
C           SFF:  NDF1 X NDF1
C           TS1:  NDA1 X NDB1
C           TS2:  NDA1 X NDF1
C           TS3:  NDA1 X NDA1
C           TS4:  NDA1 X NDF1
C
C --- NOTE: ARRAYS IN EIGENV CORRESPONDING TO TS3 AND TS4 MUST BE
C           DIMENSIONED SEPARATELY AS FOLLOWS:
C
C           FREQ:  (NDA1*NDA1/4) X 4
C           LAMBDA:  (NDA1*NDF1)
C
      NDA1=126
      NDB1=45
      NDF1=42
C
      REWIND 12
C DATA
      CALL COL1
      READ(5,*) NSUBS
C
      KFF=OLDNFT=OLDNM=NM=0
      OLDB1=1
      OLDNJBT=OLDNJAF=NJWALM=0
C
      ISM=0
  200 ISM=ISM+1
      SWT=0.
      REWIND 10
      REWIND 11
C
C-----BEGIN SUBSTRUCTURE ASSEMBLY CYCLE
C
      IF(ISM.NE.2)GO TO 206
C
C --- READ IN MASS OPTION CODE (MOPT) FOR ASSEMBLAGE OF STRUCTURE MASS
C     MATRIX:  MOPT = 1 FOR CONSISTENT MASS (CM);  MOPT = 2 FOR
C     ASSEMBLED LUMPED MASS (ALM)
C DATA
      CALL COL1
      READ(5,*) MOPT
      PRINT 205,MOPT
  205 FORMAT(*1START MASS ASSEMBLY PROCEDURE*,/,80(*0*),/,*0MOPT =*,I3,
     #  *    (MOPT=1 FOR CM, MOPT=2 FOR ALM)*)
C
  206 CONTINUE
C
      DO 100 JSU=1,NSUBS
C
C --- 1.  READ IN SUBSTRUCTURE DATA
C
      CALL SUDATA(JSU)
C
      IF(JSU.NE.1)GO TO 101
      DO 102 I=1,NDA1
      DO 103 J=1,NDA1
  103 SAA(I,J)=0.
      DO 104 LL=1,NDB1
  104 SAB(I,LL)=0.
      DO 105 JJJ=1,NDF1
  105 SAF(I,JJJ)=0.
  102 CONTINUE
      DO 106 I=1,NDB1
      DO 107 J=1,NDB1
  107 SBB(I,J)=0.
      DO 108 LL=1,NDF1
```

```
  108 SBF(I,LL)=0.
  106 CONTINUE
      DO 109 I=1,NDF1
      DO 109 J=1,NDF1
  109 SFF(I,J)=0.
  101 CONTINUE
C
C --- 2.  ASSEMBLE SUBSTRUCTURE STIFFNESS (ISM=1) AND MASS (ISM=2)
C         ARRAYS AND OVERLAY CURRENT SUBSTRUCTURE ON RESIDUALS OF
C         PREVIOUS SUBSTRUCTURE
C
      CALL OVRLAY(JSU)
C
C --- 3.  ELIMINATE A-TYPE DEGREES OF FREEDOM
C
      CALL FELIM(JSU)
C
C --- 4.  SHIFT RESIDUALS IN PREPARATION FOR NEXT OVERLAY
C
      CALL SHIFT(JSU)
C
C-----CLEAR ARRAYS SAB, SBB, AND SBF IN PREP. FOR NEXT SUBSTRUCTURE
C
      DO 110 I=1,NDB1
      DO 111 J=1,NDA1
  111 SAB(J,I)=0.
      DO 110 II=1,NDB1
  110 SBB(I,II)=0.
      DO 112 I=1,NDB1
      DO 112 J=1,NDF1
  112 SBF(I,J)=0.
C
  100 CONTINUE
C
      WRITE(12) ((SFF(I,J),J=1,KFF),I=1,KFF)
C
C-----RETURN TO FORM MASS MATRIX FOR ISM=2 CASE
C
      IF(ISM.LT.2)GO TO 200
C
C-----PRINT OUT STIFFNESS AND MASS MATRICES FOR MASTER DEGREES OF
C     FREEDOM AND COMPUTE FREQUENCIES AND MODE SHAPES
C
      REWIND 12
      READ(12) ((TS2(I,J),J=1,KFF),I=1,KFF)
C
      CALL EIGENV
C
C
      STOP
      END
```

```
*DECK SUDATA
      SUBROUTINE SUDATA(JSU)
      REAL L
      INTEGER EPV,SUPV,OLDNM,OLDNFT,OLDB1,OLDNJAF,OLDNJBT,SSN
C
      COMMON
     1/CBPARMS/ISM,NSUBS,NSUM,NSUDF,NSUFT,NAT,NBT,NABT,KFF,
     #          OLDNFT,OLDNM,NM,JTB1,OLDB1,NDA1,NDB1,NDF1,
     #          NJWAFT,NJWBT,NSUJTS,OLDNJAF,OLDNJBT,MOPT,NJWALM,E,UW
     2/CSNM/L(138),AX(138),CX(138),CY(138),CZ(138),JJ(138),JK(138),
     #       SMO(6,6),SWT
     3/CBNDOF/NFL(171),NBL(171),SUPV(171),EPV(6),
     #         SAA(126,126),SAB(126,45),SAF(126,42),
     #                      SBB(45,45),SBF(45,42),
     #                                 SFF(42,42),
     #         TS1(126,45),TS2(126,42),TS3(126,126),TS4(126,42)
     4/CBNJ/JAM(57),X(57),Y(57),Z(57)
C
C
C
      IF(ISM.EQ.2)GO TO 700
      PRINT 609,JSU,NSUBS,ISM
  609 FORMAT(*1*,80(*-*),/,*0SUBSTRUCTURE NUMBER *,I3,* OF*,I3,
     # /,1X,80(*_*), /,* ISM =*,I3)
C
C DATA
      CALL COL1
      READ(5,*) SSN,JTNOMK,NSUM,JTB1,NJWFT,E,UW
      NSUJTS=JTNOMK-OLDB1+1
      NJWBT=JTNOMK-JTB1+1
      NJWAFT=NSUJTS-NJWBT
      NM=NM+NSUM
      NSUDF=3*NSUJTS
      PRINT 508,NSUM,NSUJTS,NSUDF,NJWFT,NJWBT,NJWAFT,E,UW
  508 FORMAT(*0 NSUM NSUJTS NSUDF NJWFT NJWBT NJWAFT*,
     # *          E           UW*,/,2I5,I8,2I6,I7,1P2E13.5)
C
C-----JOINT COORDINATES
C
      PRINT 509
  509 FORMAT(*0JOINT COORDINATES*,/,* JOINT   X-COORD.     Y-COORD.*,
     # *     Z-COORD.*)
C
      IF(JSU.GT.1)GO TO 521
C --- SUBSTRUCTURE #1 - JOINT COORDINATES
      ISTRT=1
      GO TO 522
C --- SUBSTRUCTURES (#2,....,NSUBS) - JOINT COORDINATES
  521 ISTRT=OLDNJBT+1
      DO 523 I=1,OLDNJBT
      J=I+OLDNJAF
      X(I)=X(J)
      Y(I)=Y(J)
  523 Z(I)=Z(J)
C
  522 CONTINUE
C
      DO 524 I=ISTRT,NSUJTS
C DATA
      CALL COL1
```

```
  524 READ(5,*) K,X(K-OLDB1+1),Y(K-OLDB1+1),Z(K-OLDB1+1)
C
      K=OLDB1-1
      DO 506 I=1,NSUJTS
      K=K+1
  506 PRINT 507,K,X(K-OLDB1+1),Y(K-OLDB1+1),Z(K-OLDB1+1)
  507 FORMAT(I5,1P3E13.5)
C
C-----MASTER DEGREE-OF-FREEDOM LIST
C
      DO 501 I=1,NSUDF
  501 NFL(I)=NBL(I)=0
      I1=3*(JTB1-OLDB1+1)-2
      DO 503 I=I1,NSUDF
  503 NBL(I)=1
      IF(NJNFT.EQ.0)GO TO 5021
      PRINT 512
  512 FORMAT("0MASTER DEGREE OF FREEDOM LIST",/," JOINT  X-DIR  Y-DIR",
     # "  Z-DIR")
      DO 502 I=1,NJNFT
C
C DATA
      CALL COL1
      READ(5,*) J,NFL(3*(J-OLDB1+1)-2),NFL(3*(J-OLDB1+1)-1),
     #       NFL(3*(J-OLDB1+1))
  502 PRINT 5031,J,NFL(3*(J-OLDB1+1)-2),NFL(3*(J-OLDB1+1)-1),
     #             NFL(3*(J-OLDB1+1))
 5031 FORMAT(I5,I6,2I7)
 5021 NAT=NBT=NBFT=NSUFT=0
      DO 505 I=1,NSUDF
      IF(NFL(I).EQ.2)NBFT=NBFT+1
      IF(NFL(I).EQ.1)NSUFT=NSUFT+1
      IF((I.LT.I1).AND.(NFL(I).EQ.0))NAT=NAT+1
      IF(I.GE.I1)NBT=NBT+1
  505 CONTINUE
      KFF=KFF+NSJFT
      NSUAFT=NSUFT+NAT
      NABT=NAT+NBT
      PRINT 611,NAT,NBT,NABT,NBFT,NSUFT,I1,NSUAFT
  611 FORMAT("0 NAT  NBT  NABT NBFT NSUFT  I1  NSUAFT",/,5I5,2I6)
C
C-----MEMBER INCIDENCES AND PROPERTIES
C
      PRINT 510
  510 FORMAT("0MEMBER DESIGNATIONS AND PROPERTIES",/," MEMBER  JJ  JK",
     # "     AX            L              CX             CY          CZ",
     # 8X,"WEIGHT(KIPS)")
      WT=TWT=0.
      DO 504 I=1,NSUM
C
C DATA
      CALL COL1
      READ(5,*) J,JJ(I),JK(I),AX(I)
      JK1=JK(I)-OLDB1+1
      JJ1=JJ(I)-OLDB1+1
      XCL=X(JK1)-X(JJ1)
      YCL=Y(JK1)-Y(JJ1)
      ZCL=Z(JK1)-Z(JJ1)
      L(I)=SQRT(XCL*XCL+YCL*YCL+ZCL*ZCL)
      CX(I)=XCL/L(I)
      CY(I)=YCL/L(I)
      CZ(I)=ZCL/L(I)
      WT=UW*AX(I)*L(I)
      TWT=TWT+WT
      PRINT 511,J,JJ(I),JK(I),AX(I),L(I),CX(I),CY(I),
     # CZ(I),WT
  511 FORMAT(I5,2X,2I4,1P6E13.5)
  504 CONTINUE
      SWT=SWT+TWT
      PRINT 513,TWT
  513 FORMAT(//," TOTAL WEIGHT (KIPS) OF SUBSTRUCTURE MEMBERS =",
     # 1PE13.5)
      PRINT 514,SWT
  514 FORMAT(/," CUMULATIVE WEIGHT (KIPS) OF STRUCTURE TO THIS",
     #   " POINT =",1PE13.5,/)
C
C-----CONSTRUCT PERMUTATION VECTOR FOR SUBSTRUC. SUPV( )
C
      I1=0
      I2=NAT
      I3=NABT
      DO 300 I=1,NSUDF
      IF(NFL(I).EQ.1)GO TO 301
      IF(NBL(I).EQ.1)GO TO 302
C-----A TYPE DISPL.
      I1=I1+1
      SUPV(I)=I1
      GO TO 300
C-----B TYPE DISPL.
  302 I2=I2+1
      SUPV(I)=I2
      GO TO 300
C-----F TYPE DISPL.
  301 I3=I3+1
      SUPV(I)=I3+OLDNFT
  300 CONTINUE
C
C-----WRITE SUBSTRUCTURE PARAMETERS TO DISK FOR LATER USE IN MASS
C     ASSEMBLAGE PHASE
C
      WRITE(11) NSUM,NSUDF,KFF,NSUFT,NAT,NBT,NABT,JTB1,OLDB1,UW,
     #          NM,OLDNM,OLDNFT,AX,L,JJ,JK,NFL,SUPV,
     #          NJWAFT,NJWBT,NSUJTS,TWT
C
      RETURN
C
C-----READ SUBSTRUCTURE PARAMETERS OFF DISK FOR MASS MATRIX ASSEMBLY
C
  700 CONTINUE
      READ(11) NSUM,NSUDF,KFF,NSUFT,NAT,NBT,NABT,JTB1,OLDB1,UW,
     #         NM,OLDNM,OLDNFT,AX,L,JJ,JK,NFL,SUPV,
     #         NJWAFT,NJWBT,NSUJTS,TWT
C
C-----FOR MASS MATRIX ASSEMBLAGE (ISM=2), READ IN THE NUMBER OF JOINTS
C     WITH ADDITIONAL LUMPED MASS (NJWALM);  IF NJWALM > 0, READ IN
C     JOINT NUMBERS (JAM()) AND LUMPED MASSES (X())
C
C DATA
      CALL COL1
      READ(5,*) NJWALM
```

```
      PRINT 530,NJWALM,JSU
  530 FORMAT(*0ADD-ON LUMPED MASSES, NJWALM =*,I3,* FOR SUBSTRUCT*,
     *   *URE *,I3)
C
      WT=0.
C
      IF(NJWALM.EQ.0)GO TO 303
      DO 533 J=1,NSUJTS
  533 X(J)=0.
      PRINT 532
  532 FORMAT(*0JOINT    MASS(K-SECXSEC/IN)*)
      DO 534 J=1,NJWALM
C DATA
      CALL COL1
      READ(5,*) JAM(J),X(J)
      WT=WT+X(J)*386.4
  534 PRINT 535,JAM(J),X(J)
  535 FORMAT(I5,6X,1PE13.5)
  303 TWT=TWT+WT
      SWT=SWT+TWT
      PRINT 515,WT,TWT,SWT
  515 FORMAT(/,* TOTAL WEIGHT (KIPS) OF ADD-ON LUMPED MASSES =*,
     *  1PE13.5,/,* TOTAL WEIGHT (KIPS) OF SUBSTRUCTURE =*,E13.5,
     *  /,* CUMULATIVE WEIGHT (KIPS) OF STRUCTURE =*,E13.5,/)
C
      RETURN
      END
```

```
*DECK OVRLAY
      SUBROUTINE OVRLAY(JSU)
      REAL L
      INTEGER EPV,SUPV,OLDNM,OLDNFT,OLDB1,OLDNJAF,OLDNJBT
C
      COMMON
     1/CBPARMS/ISM,NSUBS,NSUM,NSUDF,NSUFT,NAT,NBT,NABT,KFF,
     *          OLDNFT,OLDNM,NM,JTB1,OLDB1,NDA1,NDB1,NDF1,
     *          NJWAFT,NJWBT,NSUJTS,OLDNJAF,OLDNJBT,MOPT,NJWALM,E,UM
     2/CBNM/L(130),AX(130),CX(130),CY(130),CZ(130),JJ(130),JK(130),
     *      SMD(6,6),SWT
     3/CBNDOF/NFL(171),NBL(171),SUPV(171),EPV(6),
     *         SAA(126,126),SAB(126,45),SAF(126,42),
     *                      SBB(45,45),SBF(45,42),
     *                                 SFF(42,42),
     *         TS1(126,45),TS2(126,42),TS3(126,126),TS4(126,42)
     4/CBNJ/JAM(57),X(57),Y(57),Z(57)
C
C
C --- CYCLE THRU MEMBERS AND OVERLAY SUBSTRUC. SUBMATRICES
C
      PRINT 500,JSU,ISM
  500 FORMAT(*0 START OVERLAY,  JSU =*,I3,*, ISM =*,I3,/,80(*-*))
C
      DO 400 I=1,NSUM
C
C-----FIRST CONSTRUCT ELEMENT PERM. VECTOR FROM SUPV( ) FOR MEMBER I
C
      JJ1=0
      JA=3*(JJ(I)-OLDB1+1)-3
      DO 310 J=1,3
      JA=JA+1
      JJ1=JJ1+1
  310 EPV(JJ1)=SUPV(JA)
      JB=3*(JK(I)-OLDB1+1)-3
      DO 311 J=1,3
      JB=JB+1
      JJ1=JJ1+1
  311 EPV(JJ1)=SUPV(JB)
C
C-----FORM MEMBER STIFFNESS (ISM=1) OR MASS (ISM=2) MATRIX FOR MEMBER I
C
      CALL SUBSM(I)
C
C     *****************************************************************
C     *                                                               *
C     *    OVERLAY SUBSTRUCTURE N SUBMATRICES ON RESIDUALS OF N-1     *
C     *                                                               *
C     *****************************************************************
C
      DO 312 I1=1,6
      IR=EPV(I1)
      IF(IR.LE.NAT)GO TO 313
      IF(IR.GT.NABT)GO TO 314
C-----B DISPL. TYPES: SBB, SBF SUBMATRICES
      IF(JSU.EQ.NSUBS)GO TO 312
      IR=IR-NAT
      DO 315 J1=1,6
      IC=EPV(J1)
      IF(IC.LE.NAT)GO TO 315
```

```
      K1=IC-NAT
      K2=IC-NABT
      IF(IC.GT.NABT)GO TO 319
      SBB(IR,K1)=SBB(IR,K1)+SND(I1,J1)
      GO TO 315
  319 SBF(IR,K2)=SBF(IR,K2)+SND(I1,J1)
  315 CONTINUE
      GO TO 312
C-----F DISPL. TYPES: SFF SUBMATRIX
  314 IR=IR-NABT
      DO 316 J1=1,6
      IC=EPV(J1)
      IF(IC.LE.NABT)GO TO 316
      K1=IC-NABT
      SFF(IR,K1)=SFF(IR,K1)+SND(I1,J1)
  316 CONTINUE
      GO TO 312
C-----A DISPL. TYPES: SAA, SAB, SAF SUBMATRICES
  313 DO 317 J1=1,6
      IC=EPV(J1)
      K1=IC-NAT
      K2=IC-NABT
      IF(IC.GT.NAT)GO TO 318
      SAA(IR,IC)=SAA(IR,IC)+SND(I1,J1)
      GO TO 317
  318 IF((IC.GT.NABT).OR.(JSU.EQ.NSUBS))GO TO 320
      SAB(IR,K1)=SAB(IR,K1)+SND(I1,J1)
      GO TO 317
  320 IF(IC.LE.NABT)GO TO 317
      SAF(IR,K2)=SAF(IR,K2)+SND(I1,J1)
  317 CONTINUE
C
  312 CONTINUE
C
  400 CONTINUE
C
C --- END OF LOOP ON MEMBERS
C
C-----FOR MASS MATRIX ASSEMBLAGE (ISM=2), AUGMENT ARRAYS MAA, MBB, AND
C     MFF WITH ADDITIONAL LUMPED MASS TERMS
C
      IF((NJMALM.EQ.0).OR.(ISM.EQ.1))RETURN
C
      DO 401 J=1,NJMALM
      J1=3*(JAM(J)-OLDB1+1)-2
      J2=J1+2
      DO 402 JJK=J1,J2
      IR=SUPV(JJK)
      IF(IR.LE.NAT)GO TO 403
      IF(IR.LE.NABT)GO TO 404
C --- F TYPE - ADD MASS TO MFF
      IR=IR-NABT
      SFF(IR,IR)=SFF(IR,IR)+X(J)
      GO TO 402
C --- A TYPE - ADD MASS TO MAA
  403 SAA(IR,IR)=SAA(IR,IR)+X(J)
      GO TO 402
C --- B TYPE - ADD MASS TO MBB
  404 IR=IR-NAT
      SBB(IR,IR)=SBB(IR,IR)+X(J)
  402 CONTINUE
  401 CONTINUE
C
      RETURN
      END
```

```
*DECK SUBSM
      SUBROUTINE SUBSM(I)
      REAL L
      INTEGER EPV,SUPV,OLDNM,OLDNFT,OLDB1,OLDNJAF,OLDNJBT
C
      COMMON
     1/CBPARMS/ISM,NSUBS,NSUM,NSUDF,NSUFT,NAT,NBT,NABT,KFF,
     #          OLDNFT,OLDNM,NM,JTB1,OLDB1,NDA1,NDB1,NDF1,
     #          NJNAFT,NJNBT,NSJJTS,OLDNJAF,OLDNJBT,MOPT,NJNALM,E,UM
     2/CBNM/L(138),AX(138),CX(138),CY(138),CZ(138),JJ(138),JK(138),
     #      SMD(6,6),SWT
     3/CBNDOF/NFL(171),NBL(171),SUPV(171),EPV(6),
     #         SAA(126,126),SAB(126,45),SAF(126,42),
     #                     SBB(45,45),SBF(45,42),
     #                                SFF(42,42),
     #         TS1(126,45),TS2(126,42),TS3(126,126),TS4(126,42)
     4/CBNJ/JAN(57),X(57),Y(57),Z(57)
C
C
      IF(ISM.EQ.2)GO TO 200
C
C-----ASSEMBLE THE MEMBER STIFFNESS (ISM=1) OR MASS (ISM=2) MATRIX
C     WRT STRUCTURE AXES FOR MEMBER I
C
C --- MEMBER STIFFNESS MATRIX
C
      SCM=(E*AX(I))/L(I)
      C1=CX(I)
      C2=CY(I)
      C3=CZ(I)
      SMD(1,1)=SMD(4,4)=SCM*C1*C1
      SMD(1,4)=SMD(4,1)=-SMD(1,1)
      SMD(1,2)=SMD(2,1)=SMD(4,5)=SMD(5,4)=SCM*C1*C2
      SMD(1,5)=SMD(5,1)=SMD(2,4)=SMD(4,2)=-SMD(1,2)
      SMD(1,3)=SMD(3,1)=SMD(4,6)=SMD(6,4)=SCM*C1*C3
      SMD(1,6)=SMD(6,1)=SMD(3,4)=SMD(4,3)=-SMD(1,3)
      SMD(2,2)=SMD(5,5)=SCM*C2*C2
      SMD(2,5)=SMD(5,2)=-SMD(2,2)
      SMD(2,3)=SMD(3,2)=SMD(5,6)=SMD(6,5)=SCM*C2*C3
      SMD(2,6)=SMD(6,2)=SMD(3,5)=SMD(5,3)=-SMD(2,3)
      SMD(3,3)=SMD(6,6)=SCM*C3*C3
      SMD(3,6)=SMD(6,3)=-SMD(3,3)
      GO TO 300
C
C
C --- MEMBER MASS MATRIX
C
  200 CONTINUE
      WT=UM*AX(I)*L(I)
      IF(MOPT.EQ.1)GO TO 203
C --- ASSEMBLED LUMPED MASS (ALM): MOPT = 2
      XMASS=WT/772.8
      DO 201 K=1,6
      DO 202 J=1,6
  202 SMD(K,J)=SMD(J,K)=0.
  201 SMD(K,K)=XMASS
      GO TO 300
C
C --- CONSISTENT MASS (CM): MOPT = 1
C
  203 XMASS=WT/2318.4
      DO 204 K=1,6
      DO 204 J=1,6
  204 SMD(K,J)=0.
      SMD(1,1)=SMD(2,2)=SMD(3,3)=SMD(4,4)=SMD(5,5)=SMD(6,6)=2.*XMASS
      SMD(1,4)=SMD(4,1)=SMD(2,5)=SMB(5,2)=SMD(3,6)=SMD(6,3)=XMASS
C
C
  300 RETURN
      END
```

```
*DECK FELIN
      SUBROUTINE FELIN(JSU)
C
C
C-----THIS SUBPROGRAM FORMS THE STRUCTURE STIFFNESS (OR MASS) MATRIX
C     (SFF) FROM SUBSTRUCTURE STIFFNESS (OR MASS) MATRICES, IN SERIES
C     ELIMINATION FASHION AS DESCRIBED IN CHAPTER 5 OF "COMPUTER
C     PROGRAMS FOR STRUCTURAL ANALYSIS" BY WM. WEAVER, JR. (1967).
C
C-----CALLED BY MAIN PROGRAM
C
C-----REQUIRES SUBPROGRAMS DCOMP AND SOLVE
C
C
      REAL L
      INTEGER EPV,SUPV,OLDNM,OLDNFT,OLDB1,OLDNJAF,OLDNJBT
C
      COMMON
     1/CBPARMS/ISN,NSUBS,NSUM,NSUDF,NSUFT,NAT,NBT,NABT,KFF,
     *          OLDNFT,OLDNM,NM,JTB1,OLDB1,NDA1,NDB1,NDF1,
     *          NJNAFT,NJNBT,NSUJTS,OLDNJAF,OLDNJBT,MOPT,NJMALM,E,UM
     2/CBNM/L(138),AX(138),CX(138),CY(138),CZ(138),JJ(138),JK(138),
     *      SMD(6,6),SMT
     3/CBNDOF/NFL(171),NBL(171),SUPV(171),EPV(6),
     *         SAA(126,126),SAB(126,45),SAF(126,42),
     *                      SBB(45,45),SBF(45,42),
     *                                 SFF(42,42),
     *         TS1(126,45),TS2(126,42),TS3(126,126),TS4(126,42)
     4/CBNJ/JAN(57),X(57),Y(57),Z(57)
C
C
      IF(NAT.EQ.0)RETURN
C
C-----COMPUTE TS1 = ((SAA)**-1)*SAB  AND  TS2 = ((SAA)**-1)*SAF
C     AND TS3 = -TS1'*MAA  AND  TS4 = -MAA*TS2
C
      IF(ISN.EQ.1) GO TO 221
      IF(JSU.EQ.NSUBS) GO TO 239
      READ(10) ((TS1(I,J),J=1,NBT),I=1,NAT)
 239  READ(10) ((TS2(I,J),J=1,KFF),I=1,NAT)
      IF(JSU.EQ.NSUBS) GO TO 240
      DO 223 I=1,NBT
      DO 223 J=1,NAT
      TEMP=0.
      DO 224 LL=1,NAT
 224  TEMP=TEMP-TS1(LL,I)*SAA(LL,J)
 223  TS3(I,J)=TEMP
 240  DO 225 I=1,NAT
      DO 225 J=1,KFF
      TEMP=0.
      DO 226 LL=1,NAT
 226  TEMP=TEMP-SAA(I,LL)*TS2(LL,J)
 225  TS4(I,J)=TEMP
      GO TO 222
 221  CALL DCOMP (NAT,SAA,NDA1,NDA1),RETURNS(208)
      GO TO 209
 208  PRINT 210
 210  FORMAT("0"," DECOMPOSITION OF SAA FAILS IN DCOMP")
 209  IF(JSU.EQ.NSUBS) GO TO 104
      DO 211 I=1,NBT
 211  CALL SOLVE(NAT,SAA,SAB,TS1,I,1,NBT,NDA1,NDA1,NDA1,NDB1,NDA1,NDB1)
      WRITE(10) ((TS1(I,J),J=1,NBT),I=1,NAT)
 104  CONTINUE
      DO 212 I=1,KFF
 212  CALL SOLVE(NAT,SAA,SAF,TS2,I,1,KFF,NDA1,NDA1,NDA1,NDF1,NDA1,NDF1)
      WRITE(10) ((TS2(I,J),J=1,KFF),I=1,NAT)
C
C-----COMPUTE SFF* = SFF - SAF'*TS2
C     AND MFF*=MFF-MAF'*TS2-TS2'*MAF-TS2'*TS4
C
 222  IF(ISN.EQ.1) GO TO 227
      DO 229 I=1,KFF
      DO 229 J=1,KFF
      TP=SFF(I,J)
      DO 230 LL=1,NAT
 230  TP=TP-SAF(LL,I)*TS2(LL,J)-TS2(LL,I)*SAF(LL,J)-TS2(LL,I)*TS4(LL,J)
 229  SFF(I,J)=TP
      GO TO 220
 227  DO 214 I=1,KFF
      DO 214 J=1,KFF
      TEMP=SFF(I,J)
      DO 215 LL=1,NAT
 215  TEMP=TEMP-SAF(LL,I)*TS2(LL,J)
 214  SFF(I,J)=TEMP
 220  IF(JSU.EQ.NSUBS) RETURN
C
C-----COMPUTE SBB* = SBB - SAB'*TS1
C     AND MBB*=MBB-MAB'*TS1-TS1'*MAB-TS3*TS1
C
      IF(ISN.EQ.1) GO TO 231
      DO 234 I=1,NBT
      DO 234 J=1,NBT
      TP=SBB(I,J)
      DO 235 LL=1,NAT
 235  TP=TP-SAB(LL,I)*TS1(LL,J)-TS1(LL,I)*SAB(LL,J)-TS3(I,LL)*TS1(LL,J)
 234  SBB(I,J)=TP
      GO TO 236
 231  DO 216 I=1,NBT
      DO 216 J=1,NBT
      TEMP=SBB(I,J)
      DO 217 LL=1,NAT
 217  TEMP=TEMP-SAB(LL,I)*TS1(LL,J)
 216  SBB(I,J)=TEMP
C
C-----COMPUTE SBF* = SBF - SAB'*TS2
C     AND MBF*=MBF-MAB'*TS2-TS1'*MAF-TS3*TS2
C
 236  IF(ISN.EQ.1) GO TO 232
      DO 237 I=1,NBT
      DO 237 J=1,KFF
      TP=SBF(I,J)
      DO 238 LL=1,NAT
 238  TP=TP-SAB(LL,I)*TS2(LL,J)-TS1(LL,I)*SAF(LL,J)-TS3(I,LL)*TS2(LL,J)
 237  SBF(I,J)=TP
      GO TO 233
 232  DO 218 I=1,NBT
      DO 218 J=1,KFF
      TEMP=SBF(I,J)
      DO 219 LL=1,NAT
 219  TEMP=TEMP-SAB(LL,I)*TS2(LL,J)
```

```
  210  SBF(I,J)=TEMP
C
  233  RETURN
       END
```

```
*DECK SHIFT
      SUBROUTINE SHIFT(JSU)
      REAL L
      INTEGER EPV,SUPV,OLDNM,OLDNFT,OLDB1,OLDNJAF,OLDNJBT
C
      COMMON
     1/CBPARMS/ISM,NSUBS,NSUM,NSUDF,NSUFT,NAT,NBT,NABT,KFF,
     *          OLDNFT,OLDNM,NM,JTB1,OLDB1,NDA1,NDB1,NDF1,
     *          NJWAFT,NJWBT,NSJJTS,OLDNJAF,OLDNJBT,MOPT,NJWALM,E,UM
     2/CBNM/L(138),AX(138),CX(138),CY(138),CZ(138),JJ(138),JK(138),
     *       SMD(6,6),SMT
     3/CBNDOF/NFL(171),NBL(171),SUPV(171),EPV(6),
     *         SAA(126,126),SAB(126,45),SAF(126,42),
     *                     SBB(45,45),SBF(45,42),
     *                                SFF(42,42),
     *         TS1(126,45),TS2(126,42),TS3(126,126),TS4(126,42)
     4/CBNJ/JAM(57),X(57),Y(57),Z(57)
C
C
C     ***********************************************************************
C     *                                                                     *
C     *   SHIFT RESIDUAL SUBMATRICES OF SUBSTRUCTURE N IN PREP-            *
C     *   ARATION FOR OVERLAY OF SUBSTRUCTURE N+1 SUBMATRICES               *
C     *                                                                     *
C     ***********************************************************************
C
      PRINT 605,JSU,ISM
  605 FORMAT(//,* NOW START SHIFT PROCEDURE, JSU =*,I3,*, ISM=*,I3,//)
C
      IF(JSU.EQ.NSUBS)RETURN
C
C-----CLEAR ARRAYS SAA AND SAF BEFORE SHIFTING
C
      DO 110 I=1,NDA1
      DO 111 J1=1,NDA1
  111 SAA(I,J1)=0.
      DO 112 J2=1,NDF1
  112 SAF(I,J2)=0.
  110 CONTINUE
C
      ICTR=0
      IS=3*(JTB1-OLDB1+1)-3
C
      DO 100 I=1,NBT
C
      I1=I-ICTR
      IF(NFL(I+IS).EQ.2)GO TO 101
C
C --- 1.  B TYPE IN SUBSTRUC. N BECOMES A TYPE IN N+1
C
      JCTR=0
      DO 102 J=1,NBT
      J1=J-JCTR
      IF(NFL(J+IS).EQ.2)GO TO 1021
C
C ---  (1.A.)  SHIFT SBB* TO SAA AND COMPACT SAA
C
      SAA(I1,J1)=SBB(I,J)
      GO TO 102
 1021 JCTR=JCTR+1
```

```
  102 CONTINUE
C
C ---  (1.B.)  SHIFT SBF* TO SAF (I.E. OLD F*S) AND COMPACT NO. ROWS
C              IN SAF
C
      IF(KFF.EQ.0)GO TO 100
      DO 103 JJK=1,KFF
  103 SAF(I1,JJK)=SBF(I,JJK)
      GO TO 100
C
C --- 2.  B TYPE IN SUBSTRUC. N BECOMES F TYPE IN N+1
C
  101 ICTR=ICTR+1
      KFF1=KFF+ICTR
      KCTR=0
      DO 200 K=1,NBT
      K1=K-KCTR
      IF(NFL(K+IS).EQ.2)GO TO 2001
C
C ---  (2.A.)  SHIFT SBB* TO SAF (I.E. NEW F*S) AND COMPACT NO. ROWS
C              IN SAF
C
      SAF(K1,KFF1)=SBB(I,K)
      GO TO 200
 2001 KCTR=KCTR+1
  200 CONTINUE
C
C ---  (2.B.)  SHIFT SBB* TO SFF* (I.E. NEW F*S) THEREBY ADDING NEW
C              COUPLING TERMS AMONG NEW F TYPES
C
      KT=KFF
      DO 201 KK=1,NBT
      IF(NFL(KK+IS).NE.2)GO TO 201
      KT=KT+1
      SFF(KFF1,KT)=SBB(I,KK)
  201 CONTINUE
C
C ---  (2.C.)  SHIFT SBF* (OLD F*S) TO SFF* (NEW F*S) THEREBY COUPLING
C              OLD AND NEW F TYPES
C
      IF(KFF.EQ.0)GO TO 100
      DO 202 LL=1,KFF
      SFF(KFF1,LL)=SFF(LL,KFF1)=SBF(I,LL)
  202 CONTINUE
C
C
  100 CONTINUE
C
      OLDNM=NM
      OLDB1=JTB1
      OLDNJBT=NJNBT
      OLDNJAF=NJNAFT
      OLDNFT=KFF
      PRINT 606,OLDNM,OLDB1,OLDNFT,OLDNJBT,OLDNJAF
  606 FORMAT(*0 OLDNM OLDB1 OLDNFT OLDNJBT OLDNJAF*,/,I5,2I6,2I8)
C
      RETURN
      END
```

```
*DECK DCOMP
      SUBROUTINE DCOMP (N,A,MA(1,MAX2),RETURNS(JBJ)
C
C-----THIS SUBPROGRAM DECOMPOSES THE SYMMETRIC, POSITIVE-DEFINITE
C     MATRIX A USING THE CHOLESKY SQUARE ROOT METHOD AS DISCUSSED
C     ON PAGE 56 OF 'COMPUTER PROGRAMS FOR STRUCTURAL ANALYSIS,' (1967)
C     BY WM. WEAVER, JR.,  MATRIX A IS NXN, BUT MAY BE DIMENSIONED FOR
C     MAX1 X MAX2 IN THE CALLING PROGRAM.
C
C
      DIMENSION A(MAX1,MAX2)
C
      DO 101 I=1,N
      DO 101 J=I,N
      SUM=A(I,J)
      K1=I-1
      IF(I.EQ.1) GO TO 1
      DO 102 K=1,K1
 102  SUM=SUM-A(K,I)*A(K,J)
  1   IF(J.NE.I) GO TO 2
      IF(SUM.LE.0.) RETURN JBJ
      TEMP=1.0/SQRT(SUM)
      A(I,J)=TEMP
      GO TO 101
  2   A(I,J)=SUM*TEMP
 101  CONTINUE
C
      RETURN
      END
```

```
*DECK SOLVE
      SUBROUTINE SOLVE (N,U,B,X,II,IC,NN,MU1,MU2,MB1,MB2,MX1,MX2)
C
C-----THIS SUBPROGRAM SOLVES THE SYSTEM OF SIMULTANEOUS EQUATIONS,
C     A * X = B, IN A TWO-STEP PROCESS AS DISCUSSED ON PAGE 57 OF
C     'COMPUTER PROGRAMS FOR STRUCTURAL ANALYSIS' BY WM. WEAVER, JR.
C     (1967).  SUBPROGRAM 'DCOMP' PROVIDES THE DECOMPOSED VERSION OF
C     MATRIX A (STORED IN UPPER TRIANGULAR MATRIX U) NEEDED IN THE
C     SOLUTION PROCESS.  X IS THE VECTOR (OR MATRIX) OF UNKNOWNS AND
C     B IS THE VECTOR (OR MATRIX) OF KNOWN QUANTITIES.
C          SUBPROGRAM 'SOLVE' IS MODIFIED HERE SO THAT IT CAN BE USED
C     AS THE OBJECT OF A DO STATEMENT WHERE II IS THE DO VARIABLE.
C     THUS, THE MATRIX OF UNKNOWNS, X(N,NN), CAN BE OBTAINED BY CALLING
C     'SOLVE' NN TIMES.
C     NOTE: IF IC=1, MATRIX B IS USED AS IS; IF IC=2, THE TRANSPOSE
C           OF MATRIX B IS USED.
C
C-----ARRAY SIZES:  U(N X N), B(N X NN), X(N X NN)
C
C*****MU1.....MX2 ARE THE DIMENSIONS OF ARRAYS U, B, AND X
C     IN THE CALLING PROGRAM.
C
C
      DIMENSION U(MU1,MU2),B(MB1,MB2),X(MX1,MX2)
C
      DO 101 I=1,N
      SUM=B(I,II)
      IF(IC.EQ.2) SUM=B(II,I)
      K1=I-1
      IF(I.EQ.1) GO TO 101
      DO 102 K=1,K1
 102  SUM=SUM-U(K,I)*X(K,II)
 101  X(I,II)=SUM*U(I,I)
C
      DO 103 I1=1,N
      I=N-I1+1
      SUM=X(I,II)
      K2=I+1
      IF(I.EQ.N) GO TO 103
      DO 104 K=K2,N
 104  SUM=SUM-U(I,K)*X(K,II)
 103  X(I,II)=SUM*U(I,I)
C
      RETURN
      END
```

```
*DECK MATPRT
      SUBROUTINE MATPRT(M,N,A,MAX1,MAX2)
C
C-----THIS SUBPROGRAM PRINTS AN M BY N MATRIX 9 COLUMNS AT A TIME
C
C*****MAX1 AND MAX2 ARE THE DIMENSIONS OF THE ARRAY "A" IN THE CALLING
C     PROGRAM.
C
C-----CALLED BY MAIN PROGRAM
C
C
      REAL A(MAX1,MAX2)
      INTEGER RTCOL
C
  601 FORMAT (" ",I3,1X,1P6E13.5)
  602 FORMAT ("0COLUMN:",I4,3X,6(I10,3X))
  603 FORMAT(" ------",/,"  ROW")
      NPAGES = (N-1)/6 + 1
      DO 101 I=1,NPAGES
      LTCOL = 6*(I-1) + 1
      RTCOL = 6*I
      IF (RTCOL.GT.N) RTCOL=N
      PRINT 602,(K,K=LTCOL,RTCOL)
      PRINT 603
      DO 101 J=1,M
  101 PRINT 601,J,(A(J,K),K=LTCOL,RTCOL)
      RETURN
      END
```

```
*DECK EIGENV
      SUBROUTINE EIGENV
C
C-----SUBPROGRAM TO SOLVE THE NON-STANDARD FORM OF THE EIGENVALUE
C     PROBLEM:
C                 M*X = (1/P**2)*S*X
C
C     OR          A*X = LAMBDA*B*X
C
C     PARAMETERS:
C       ON INPUT -   N=NO. DOF
C                    A=MASS MATRIX
C                    B=STIFFNESS MATRIX
C       ON OUTPUT -  SA=PSQ (SQUARES OF NAT. CIRC. FREQUENCIES)
C                    A=MASS MATRIX
C                    S=STIFFNESS MATRIX
C                    X=XN1 (INVERSE OF XN)
C                    B=XN (MODAL MATRIX NORMALIZED WRT MASS MATRIX)
C
      REAL LAMBDA
      INTEGER EPV,SUPV,OLDNM,OLDNFT,OLDB1,OLDNJAF,OLDNJBT
C
      COMMON
     1/CBPARMS/ISN,NSUBS,NSUM,NSUDF,NSUFT,NAT,NBT,NABT,KFF,
     #          OLDNFT,OLDNM,NM,JTB1,OLDB1,NDA1,NDB1,NDF1,
     #          NJNAFT,NJNBT,NSJJTS,OLDNJAF,OLDNJBT,MOPT,NJNALM,E,UM
     3/CBNDOF/NFL(171),NBL(171),SUPV(171),EPV(6),
     #         SAA(126,126),SAB(126,45),SAF(126,42),
     #                    SBB(45,45),SBF(45,42),
     #                               A(42,42),
     #         X(126,45),B(126,42),FREQ(3969,4),LAMBDA(5292)
C
C
      N=KFF
C
  101 FORMAT(*1FREQUENCIES AND MODE SHAPES*,/,
     # * ----------------------------*)
  102  FORMAT(*0*,* SYSTEM MASS MATRIX*)
  100  FORMAT(*0*,* SYSTEM STIFFNESS MATRIX*)
  110  FORMAT(*0*,* CIRCULAR FREQUENCIES (RADIANS/SEC.)*)
  111  FORMAT(*0*,* NATURAL FREQUENCIES (HERTZ)*)
  112  FORMAT(*0*,* NATURAL PERIODS (SECONDS)*)
  113  FORMAT(*0*,* MODAL MATRIX XM (SCALED TO MAKE LARGEST ENTRY 1.0)*)
  104  FORMAT(1P9E13.5)
  105  FORMAT(*   ERROR IN AXLBX  *)
      PRINT 101
      PRINT 102
       CALL MATPRT(N,N,A,NDF1,NDF1)
      PRINT 100
       CALL MATPRT(N,N,B,NDA1,NDF1)
      CALL AXLBX(N),RETURNS(106)
       GO TO 107
  106  PRINT 105
  107  CONTINUE
       DO 103 I=1,N
       FREQ(I,1)=SQRT(1/LAMBDA(I))
       FREQ(I,2)=FREQ(I,1)/6.283185308
  103  FREQ(I,3)=1./FREQ(I,2)
       PRINT 110
       PRINT 104,(FREQ(I,1),I=1,N)
       PRINT 111
       PRINT 104,(FREQ(I,2),I=1,N)
       PRINT 112
       PRINT 104,(FREQ(I,3),I=1,N)
       PRINT 113
       CALL MATPRT(N,N,X,NDA1,NDB1)
C
       RETURN
       END
```

```
*DECK AXLBX
      SUBROUTINE AXLBX(N),RETURNS(JJ)
C
C-----SOLVES EIGENPROBLEM A*X = LAMBDA*B*X
C     A SYMMETRIC, B SYMMETRIC POSITIVE DEFINITE
C     INPUT: A AND B (ONLY DIAGONAL AND UPPER TRIANGLE NEEDED)
C     OUTPUT: LAMBDA AND X
C     ALTERED: DIAGONAL AND LOWER TRIANGLE OF A AND B
C     ERROR RETURN: B IS NOT POSITIVE DEFINITE OR TQL2 DID NOT CONVERGE
C     REFERENCES: NUMERERISCHE MATHEMATIK, VOL. 11, PP. 99, 181 AND 293.
C
C
C-----CALLED BY SUBPROGRAM EIGENV
C
C-----REQUIRES SUBPROGRAMS TRED2 AND TQL2
C
C
      REAL LAMBDA,E1(256)
      COMMON
     1/CBPARMS/ISM,NSUBS,NSUM,NSUDF,NSUFT,NAF,NBT,NABT,KFF,
     &          OLDMFT,OLDNM,NM,JTB1,OLDB1,NDA1,NDB1,NDF1,
     &          NJNAFT,NJNBT,NSJJTS,OLDNJAF,OLDNJBT,NOPT,NJNALM,E,UN
     3/CBNDOF/NFL(171),NBL(171),SUPV(171),EPV(6),
     &         SAA(126,126),SAB(126,45),SAF(126,42),
     &                      SBB(45,45),SBF(45,42),
     &                                 A(42,42),
     &         X(126,45),B(126,42),FREQ(3969,4),LAMBDA(5292)
C
      DO 4 I=1,N
      DO 4 J=I,N
      S=B(I,J)
        IF(I.EQ.1) GO TO 2
        I1=I-1
        DO 1 K=1,I1
    1     S=S-B(I,K)*B(J,K)
    2   IF(J.NE.I) GO TO 3
        IF(S.LE.0. ) GO TO 13
        T=SQRT(S)
        B(I,I)=T
        GO TO 4
    3   B(J,I)=S/T
    4 CONTINUE
      DO 6 I=1,N
      DO 6 J=I,N
        S=A(I,J)
        IF(I.EQ.1) GO TO 6
        I1=I-1
        DO 5 K=1,I1
    5     S=S-B(I,K)*A(J,K)
    6 A(J,I)=S/B(I,I)
      DO 10 J=1,N
      DO 10 I=J,N
        S=A(I,J)
        IF(I.EQ.J) GO TO 8
        I1=I-1
        DO 7 K=J,I1
    7     S=S-A(K,J)*B(I,K)
    8   IF(J.EQ.1) GO TO 10
        J1=J-1
        DO 9 K=1,J1
    9     S=S-A(J,K)*B(I,K)
   10 A(I,J)=S/B(I,I)
C
      CALL TRED2(NDF1,NDA1,N,2.44E-63,A,LAMBDA,E1,X)
      CALL TQL2(NDA1,N,2.22E-13,LAMBDA,E1,X,IERR)
      IF(IERR.NE.0) GO TO 13
C
      DO 12 J=1,N
      DO 12 IBACK=1,N
      I=N+1-IBACK
        S=X(I,J)
        IF(I.EQ.N) GO TO 12
        I1=I+1
        DO 11 K=I1,N
   11     S=S-B(K,I)*X(K,J)
   12 X(I,J)=S/B(I,I)
C-----SCALE EIGENVECTORS TO MAKE LARGEST ENTRY = 1.0
      DO 301 J=1,N
      BIG=X(1,J)
      DO 302 I=2,N
      IF(ABS(X(I,J)).LE.ABS(BIG))GO TO 302
      BIG=X(I,J)
  302 CONTINUE
      DO 303 I=1,N
  303 X(I,J)=X(I,J)/BIG
  301 CONTINUE
      RETURN
   13 RETURN JJ
      END
```

```
*DECK TQL2
      SUBROUTINE TQL2 (NM,N,MACHEP,D,E,Z,ERROR)
C
      REAL MACHEP,D(N),E(N),Z(NM,N)
      INTEGER ERROR
C
C-----THIS SUBPROGRAM IS A TRANSLATION OF THE ALGOL PROCEDURE TQL2,
C     NUM.MATH.11, 293-306(1968) BY BOWDLER, MARTIN, REINSCH, AND
C     WILKINSON.
C
C     THIS SUBPROGRAM USES QL TRANSFORMATIONS TO FIND THE
C     EIGENVALUES AND EIGENVECTORS OF A TRIDIAGONAL MATRIX,
C     GIVEN WITH ITS DIAGONAL ELEMENTS IN THE ARRAY D(N)
C     AND ITS SUBDIAGONAL ELEMENTS IN THE LAST N-1 ELEMENTS
C     OF THE ARRAY E(N).
C
C     THE EIGENVALUES ARE OVERWRITTEN ON THE DIAGONAL ELEMENTS
C     IN THE ARRAY D IN ASCENDING ORDER.
C
C     THE EIGENVECTORS ARE FORMED IN THE ARRAY Z(N,N), OVERWRITING
C     THE ACCUMULATED TRANSFORMATIONS AS SUPPLIED BY TRED2.
C
C     IF THE TRIDIAGONAL MATRIX IS PRIMARY DATA (THUS, TRED2
C     HAS NOT BEEN USED), Z SHOULD BE PRESET TO THE IDENTITY
C     MATRIX.
C
C     MACHEP IS THE RELATIVE MACHINE PRECISION.  MACHEP SHOULD
C     BE SET TO 2**(-52) FOR LONG FORM ARITHMETIC ON S/360.
C
C     THE PROCEDURE FAILS (ERROR IS SET TO THE INDEX OF THE
C     EIGENVALUE FOR WHICH FAILURE OCCURRED) IF ANY EIGENVALUE
C     TAKES MORE THAN 30 ITERATIONS.
C
C     NM MUST BE SET TO THE ROW DIMENSION OF TWO-DIMENSIONAL ARRAY
C     PARAMETERS AS DECLARED IN THE CALLING PROGRAM DIMENSION STATEMENT.
C
C     TRANSLATED BY V. KLEMA, ARGONNE NATIONAL LABORATORY, NOV., 1968.
C     MODIFIED BY B. GARBOW, JAN. 1971.
C
C-----CALLED BY SUBPROGRAM "AXLBX"
C
C
C     ------------------------------------------------------------------
C
      ERROR = 0
      IF (N .EQ. 1) GO TO 1001
C
      DO 100 I = 2, N
  100 E(I-1) = E(I)
C
      F = 0.0
      B = 0.0
      E(N) = 0.0
C
      DO 240 L = 1, N
         J = 0
         H = MACHEP * (ABS(D(L)) + ABS(E(L)))
         IF (B .LT. H) B = H
C
C C:C  LOOK FOR SMALL SUB-DIAGONAL ELEMENT
C
         DO 110 M = L, N
         IF (ABS(E(M)) .LE. B) GO TO 120
 110  CONTINUE
C
  120    IF (M .EQ. L) GO TO 220
  130    IF (J .EQ. 30) GO TO 1000
         J = J + 1
C
CCCCC: FORM SHIFT
CCC:CCC:
         P = (D(L+1) - D(L)) / (2.0  * E(L))
         R = SQRT(P * P + 1.0 )
         H = D(L) - E(L) / (P + SIGN(R,P))
C
         DO 140 I = L, N
  140    D(I) = D(I) - H
C
         F = F + H
C
CCCCC: QL TRANSFORMATION
CCCCC
         P = D(M)
         C = 1.0
         S = 0.0
         MML = M - L
C
CCCCC: FOR I=M-1 STEP -1 UNTIL L DO -- CCC:CCCCC:
         DO 200 II = 1, MML
            I = M - II
            G = C * E(I)
            H = C * P
            IF (ABS(P) .LT. ABS(E(I))) GO TO 150
            C = E(I) / P
            R = SQRT(C * C +1.0 )
            E(I+1) = S * P * R
            S = C / R
            C = 1.0  / R
            GO TO 160
  150       C = P / E(I)
            R = SQRT(C * C + 1.0  )
            E(I+1) = S * E(I) * R
            S = 1.0  / R
            C = C / R
  160       P = C * D(I) - S * G
            D(I+1) = H + S*(C*G + S*D(I))
C
CCCCC: FORM VECTOR CCC:CCCCC:
            DO 180 K = 1, N
               H = Z(K,I+1)
               Z(K,I+1) = S * Z(K,I) + C *  H
               Z(K,I) = C * Z(K,I) - S * H
  180       CONTINUE
C
  200    CONTINUE
C
         E(L) = S * P
         D(L) = C * P
         IF (ABS(E(L)) .GT. B) GO TO 130
  220    D(L) = D(L) + F
```

```
  240 CONTINUE
C
CCCCC: ORDER EIGENVALUES AND EIGENVECTORS
CC
      NM1 = N - 1
      DO 300 I = 1, NM1
         K = I
         P = D(I)
         IP1 = I + 1
C
         DO 260 J = IP1, N
            IF (D(J) .LE. P) GO TO 260
            K = J
            P = D(J)
  260    CONTINUE
C
         IF (K .EQ. I) GO TO 300
         D(K) = D(I)
         D(I) = P
C
         DO 280 J = 1, N
            P = Z (J,I)
            Z(J,I) = Z(J,K)
            Z (J,K) = P
  280    CONTINUE
C
  300 CONTINUE
C
      GO TO 1001
C
CCCCC: FAIL EXIT STATEMENT IS 1000
CCCCC
 1000 ERROR = L
 1001 RETURN
C
CCCCC: LAST CARD OF TQL2
CCCCC
      END
```

```
*DECK TRED2
      SUBROUTINE TRED2  (NM1,NM2,N,TOL,A,D,E,Z)
C
      REAL A(NM1,N),D(N),E(N),Z(NM2,N)
C
C-----THIS SUBPROGRAM REDUCES THE GIVEN LOWER TRIANGLE OF A
C     SYMMETRIC MATRIX   STORED IN  THE ARRAY A(N,N) TO
C     TRIDIAGONAL FORM USING HOUSEHOLDER"S REDUCTION.
C     THE DIAGONAL OF THE RESULT IS STORED IN THE ARRAY D(N)
C     AND THE SUB-DIAGONAL IN THE LAST N-1 ELEMENTS OF THE ARRAY
C     E(N)  (WITH THE ADDITIONAL ELEMENT E(1)=0).
C     THE TRANSFORMATION MATRICES ARE ACCUMULATED IN THE ARRAY Z(N,N).
C     TOL IS A TOLERANCE FOR CHECKING IF THE TRANSFORMATION IS VALID.
C     TOL SHOULD BE SET TO 10.**(-60) FOR LONG FORM ARITHMETIC ON S360.
C     NM1 AND NM2 ARE THE ROW DIMENSIONS OF THE TWO DIMENSIONAL ARRAYS
C     "A" AND "X" AS DECLARED IN THE CALLING PROGRAM DIMENSION STATE-
C     MENT. THE ARRAY A IS LEFT UNALTERED UNLESS THE ACTUAL PARAMETERS
C     CORRESPONDING TO A AND Z ARE IDENTICAL.
C     THIS SUBPROGRAM IS A TRANSLATION OF THE ALGOL PROCEDURE TRED2,
C     NUM. MATH. 11,181-195(1968) BY MARTIN, REINSCH, AND WILKINSON.
C
C
C-----CALLED BY SUBPROGRAM "AKLBX"
C
      DO 100 I=1,N
      DO 100 J=1,I
  100 Z(I,J)=A(I,J)
      IF (N.EQ.1) GO TO 1015
C     FOR I= N STEP -1 UNTIL 2 DO --
      DO 1011 II=2,N
      I = N+2 - II
      L=I-2
      F =Z(I,I-1)
      G=0.0
      IF(L.LT.1) GO TO 103
      DO 102 K = 1,L
  102 G= G + Z(I,K)*Z(I,K)
  103 H = G + F*F
C     IF G IS TOO SMALL FOR ORTHOGONALITY TO BE GUARANTEED, THE
C     TRANSFORMATION IS SKIPPED.
      IF (G.GT. TOL) GO TO 104
      E(I)= F
      H = 0.0
      GOTO 101
  104 L=L+1
      G=SQRT (H)
      IF (F.GE.0.0 ) G=-G
      E(I) = G
      H = H-F*G
      Z(I,I-1) = F-G
      F = 0.0
      DO 105 J= 1,L
      Z(J,I) = Z(I,J)/H
      G=0.0
C     FORM ELEMENT OF A*U
      DO 106 K = 1,J
  106 G = G + Z(J,K) * Z(I,K)
      JP1 = J+1
      IF (L .LT. JP1) GOTO 9108
      DO 108 K = JP1,L
```

```
  108 G = G + Z(K,J) * Z(I,K)                                          TRE00600
C     FORM ELEMENT OF P                                                TRE00610
 9108 E(J) = G/H                                                       TRE00620
      F = F + G*Z(J,I)                                                 TRE00630
  105 CONTINUE                                                         TRE00640
C     FORM K -- SEE ALGOL TRED2                                        TRE00650
      HH = F/(H+H)                                                     TRE00660
C     FORM REDUCED A                                                   TRE00670
      DO 109 J = 1,L                                                   TRE00680
      F = Z (I,J)                                                      TRE00690
      E(J) = E (J) - HH * F                                            TRE00700
      G = E(J)                                                         TRE00710
      DO 109 K = 1,J                                                   TRE00720
  109 Z(J,K) = Z(J,K)- F * E(K) - G * Z(I,K)                           TRE00730
C     101 CORRESPONDS TO SKIP                                          TRE00740
  101 D(I) = H                                                         TRE00750
 1011 CONTINUE                                                         TRE00760
 1015 D(1) = 0.0                                                       TRE00770
      E(1) = 0.0                                                       TRE00780
C     ACCUMULATION OF TRANSFORMATION MATRICES                          TRE00790
      DO 201 I = 1,N                                                   TRE00800
      L = I - 1                                                        TRE00810
      IF (D(I) .EQ. 0.0 ) GO TO 202                                    TRE00820
      DO 203 J = 1,L                                                   TRE00830
      G = 0.0                                                          TRE00840
      DO 204 K = 1,L                                                   TRE00850
  204 G = G + Z(I,K)*Z(K,J)                                            TRE00860
      DO 203 K = 1,L                                                   TRE00870
  203 Z(K,J) = Z (K,J) - G*Z (K,I)                                     TRE00880
  202 D(I) = Z(I,I)                                                    TRE00890
      Z(I,I) = 1.0                                                     TRE00900
      IF (L .LT. 1) GOTO 201                                           TRE00910
      DO 205 J = 1,L                                                   TRE00920
      Z(I,J) = 0.0                                                     TRE00930
  205 Z(J,I) = 0.0                                                     TRE00940
  201 CONTINUE                                                         TRE00950
      RETURN                                                           TRE00960
      END                                                              TRE00970
```

```
*DECK COL1                                                             COL00000
      SUBROUTINE COL1                                                  COL00010
C                                                                      COL00020
C     SUBPROGRAM TO PERMIT COMMENT CARDS IN DATA FILES: COMMENTS MUST  COL00030
C     BEGIN WITH A "$" IN COLUMN 1                                     COL00040
C                                                                      COL00050
   10 READ(5,20)  APLHA                                                COL00060
   20 FORMAT(A1)                                                       COL00070
      IF(APLHA.NE."$")GO TO 30                                         COL00080
      GO TO 10                                                         COL00090
   30 BACKSPACE 5                                                      COL00100
      RETURN                                                           COL00110
      END                                                              COL00120
```

Table C.1 - Data Input Guide for Program DLETR

| Data Item | Number of Cards | Description |
|---|---|---|
| TINC,TMAX, NXDATA,NYDATA, NZDATA,NPPC, TINTEX | 1 | TINC is integration time interval.<br>TMAX is duration of forcing function.<br>NXDATA,NYDATA and NZDATA are number of points describing the forcing functions in the X, Y and Z directions respectively.<br>NPPC is the number of forcing function points read off tape at a time.<br>TINTEX is the time interval between forcing function points when it is read off tape. |
| KX | 1 | KX = 1 if the forcing function in the X-direction is on tape. |
| T(J),F(J) | As Necessary | Only if KX ≠ 1 time points and forcing function values input for the X-direction. |
| F(I) | On Tape | Only if KX = 1 forcing function values in the X-direction at time interval TINTEX. |
| T(J),F(J) | As Necessary | Time points and forcing function values input for the Y-direction. |
| T(J),F(J) | As Necessary | Time points and forcing function values input for the Z-direction. |
| NDOFS | On Tape | Number of degrees of freedom. |
| NRA(J) | As Necessary | Arrangement and order of degrees of freedom<br>X → 1<br>Y → 2<br>Z → 3 |
| LF(J) | As Necessary | Load factors at dynamic degrees of freedom. |
| D(J,1),DV(J) | NDOFS | Initial displacement and velocity at degrees of freedom |

Table C.1 - Data Input Guide for Program DLETR (continued)

| Data Item | Number of Cards | Description |
|---|---|---|
| M(I,J) | On Tape | The mass matrix |
| S(I,J) | On Tape | The stiffness matrix |
| C(I,J) | On Tape | The damping matrix |

```
*DECK DLETR
      PROGRAM DLETR(INPUT,OUTPUT,SANDM,CDAMP,ACCELX,DISPC1,DISP2P,DISP52
     $,TAPE23=SANDM,TAPE24=DISP2P,TAPE25=DISP52,
     $TAPE21=CDAMP,TAPE22=ACCELX,TAPE23=DISPC1,TAPE6=OUTPUT)
      DIMENSION F(200),T(200),FXI(400),FYI(400),FZI(400),TI(400),NRA(63)
     $,S(63,63),C(63,63),A(63),D(63,400),DV(63),DA(63),SS(63,63),TM(63,
     $63),TEMP1(63),TEMP2(63),TA(63),P(63),Q(63)
      REAL M(63,63),LF(63)
C
      MAX=63
      MAXI=400
      REWIND 23
C
C ----- INPUT FOR INTERP
C
C DATA
      READ*,TINC,TMAX,NXDATA,NYDATA,NZDATA,NPPC,TINTEX
C
C ----- READ INFORMATION ABOUT INPUT RECORD
C         KX=1 IF FX IS ON TAPE
C         KX=3 IF FX IS ON FILE
C
C DATA
      READ*,KX
C
C ----- FX INPUT RECORD
C
C DATA
      NN=NPPC
      NXTOT=1
      PRINT 400,KX
  400 FORMAT(5X,I8)
      IF(KX.EQ.1) GO TO 201
      READ*,(T(J),F(J),J=1,NXDATA)
      GO TO 202
 201  REWIND 22
      WRITE(23) NPPC,TINC,TMAX,TINTEX
      WRITE(24) NPPC,TINC,TMAX,TINTEX
      WRITE(25) NPPC,TINC,TMAX,TINTEX
 300  NXTOT=NXTOT+NN-1
      IF(NXTOT.GT.NXDATA) NN=NN+NXDATA-NXTOT
      NO=2
      IF(NXTOT.EQ.NN) NO=1
      IF(NXTOT.GT.NN) F(1)=FEND
      READ(22,*) (F(I),I=NO,NN)
      FEND=F(NN)
      IF(NXTOT.NE.NN) GO TO 401
      PRINT 40,TINTEX
  401 CONTINUE
      TALL=0.
      DO 203 I=1,NN
      T(I)=TALL
  203 TALL=TALL+TINTEX
C
C ------ PRINT FX INPUT RECORD
C
  202 PRINT 20
   20 FORMAT(*1*,*INPUT RECORD PARAMETERS FOR FX*,//)
      PRINT 21,TMAX,NXDATA,TINC
   21 FORMAT(*     DURATION(SEC)=*F10.3,/,*     NO. OF VALUES=*I8,/,*
     $  INTERPOLATION TIME INTERVAL(SEC)=*F7.4,///)
      PRINT 22
   22 FORMAT(* INPUT RECORD - TIME VS.ACCEL,VEL, OR DISPL*/)
      PRINT 23,(T(I),F(I),I=1,NN)
   23 FORMAT(5(F8.3,F12.5,2X))
C
C ----- INTERP CALLED TO DIGITIZE FX RECORD AT EQUAL TIME INTERVALS
C
      TTAX=(NN-1)*TINTEX
      CALL INTERP(F,T,TINC,TTAX,NN,NPTS,FXI,TI,TTOT)
C
C ----- DIGITIZED RECORD OF FX PRINTED
C
      PRINT 33
   33 FORMAT(//,* EQUAL TIME INTERVAL DIGITIZATION OF INPUT RECORD FX*/)
      PRINT 34,TTOT,NPTS
   34 FORMAT(*     DURATION(SEC)=*F10.3,/,*     NO. OF VALUES=*I8,///)
      PRINT 35
   35 FORMAT(* INTERPOLATED RECORD - TIMES AND VALUES*,/)
      PRINT 36,(TI(J),FXI(J),J=1,NPTS)
   36 FORMAT(8(F12.5,2X))
C
C ----- FY INPUT RECORD
C
C DATA
      IF(NXTOT.GT.NN) GO TO 301
      READ*,(T(J),F(J),J=1,NYDATA)
C
C ----- PRINT OUT FY INPUT RECORD
C
      PRINT 24
   24 FORMAT(*1*,*INPUT RECORD PARAMETERS FOR FY*,//)
      PRINT 21,TMAX,NYDATA,TINC
      PRINT 22
      PRINT 23,(T(I),F(I),I=1,NYDATA)
C
C ----- INTERP CALLED TO DIGITIZE FY RECORD AT EQUAL TIME INTERVALS
C
      CALL INTERP(F,T,TINC,TTAX,NYDATA,NPTS,FYI,TI,TTOT)
C
C ----- DIGITIZED RECORD OF FY PRINTED
C
      PRINT 37
   37 FORMAT(//,* EQUAL TIME INTERVAL DIGITIZATION OF INPUT RECORD FY*/)
      PRINT 34,TTOT,NPTS
      PRINT 35
      PRINT 36,(TI(J),FYI(J),J=1,NPTS)
C
C ----- FZ INPUT RECORD
C
C DATA
      READ*,(T(J),F(J),J=1,NZDATA)
C
C ----- PRINT OUT FZ INPUT RECORD
C
      PRINT 25
   25 FORMAT(*1*,*INPUT RECORD PARAMETERS FOR FZ*,//)
      PRINT 21,TMAX,NZDATA,TINC
      PRINT 22
      PRINT 23,(T(I),F(I),I=1,NZDATA)
```

```
C
C ----- CALL INTERP TO DIGITIZE FZ RECORD AT EQUAL TIME INTERVALS
C
      CALL INTERP(F,T,TINC,TTAX,NZDATA,NPTS,FZI,TI,TTOT)
C
C ----- DIGITIZED RECORD OF FZ PRINTED
C
      PRINT 38
   38 FORMAT(//,* EQUAL TIME INTERVAL DIGITIZATION OF INPUT RECORD FZ*/)
      PRINT 34,TTOT,NPTS
      PRINT 35
      PRINT 36,(TI(J),FZI(J),J=1,NPTS)
C
C ----- INPUT FOR CONSTRUCTION OF A
C
C DATA
      REWIND 20
      READ(20)NDOFS
      READ*,(NRA(J),J=1,NDOFS)
      READ*,(LF(J),J=1,NDOFS)
C
C ----- PRINT INFORMATION FOR CONSTRUCTION OF A
C
      PRINT 42,NDOFS
   42 FORMAT(*1*,*   NUMBER OF DEGREES OF FREEDOM = *I8,/)
      PRINT 43
   43 FORMAT(//,*   VECTOR CONTAINING INFORMATION ABOUT ORDER AND KIND O
     $F DEGREES OF FREEDOM*///)
      DO 44 I=1,NDOFS
   44 PRINT 45,NRA(I)
   45 FORMAT(10X,I8)
      PRINT 46
   46 FORMAT(///,*   VECTOR OF LOAD FACTORS*///)
      DO 47 I=1,NDOFS
   47 PRINT 48,LF(I)
   48 FORMAT(10X,F12.7)
C
C ----- INITIALIZE  D AND DV
C
C DATA
      DO 11 J=1,NDOFS
   11 READ*,D(J,1),DV(J)
C
C ----- PRINT INITIAL  D AND DV
C
      PRINT 49
   49 FORMAT(*1*,*   INITIAL VALUE OF D*//)
      DO 50 I=1,NDOFS
   50 PRINT 48,D(I,1)
      PRINT 51
   51 FORMAT(///,*   INITIAL VALUE OF DV*///)
      DO 52 I=1,NDOFS
   52 PRINT 48,DV(I)
C
C ----- READ THE STIFFNESS, MASS AND DAMPING MATRICES
C
      READ(20) ((M(I,J),J=1,NDOFS),I=1,NDOFS)
      READ(20) ((S(I,J),J=1,NDOFS),I=1,NDOFS)
      REWIND 21
      READ(21) ((C(I,J),J=1,NDOFS),I=1,NDOFS)
C
C ----- PRINT THE MASS, DAMPING AND STIFFNESS MATRICES
C
      PRINT 61
   61 FORMAT(*1*,*      THE MASS MATRIX*/)
      CALL MATPRT(NDOFS,NDOFS,M,MAX,MAX)
      PRINT 62
   62 FORMAT(*1*,*      THE DAMPING MATRIX*/)
      CALL MATPRT(NDOFS,NDOFS,C,MAX,MAX)
      PRINT 63
   63 FORMAT(*1*,*      THE STIFFNESS MATRIX*/)
      CALL MATPRT(NDOFS,NDOFS,S,MAX,MAX)
C
C ----- COMPUTE INITIAL VALUE OF DA
C
      DO 53 I=1,NDOFS
      DO 53 J=1,NDOFS
   53 TM(I,J)=M(I,J)
      L=1
      CALL CONSTA(NDOFS,MAX,NRA,LF,FXI,FYI,FZI,L,A,MAXI)
      DO 54 I=1,NDOFS
      P(I)=D(I,1)
   54 TA(I)=A(I)
      CALL MULTI(C,DV,NDOFS,MAX,TEMP1)
      CALL MULTI(S,P,NDOFS,MAX,TEMP2)
      DO 55 I=1,NDOFS
   55 TA(I)=TA(I)-TEMP1(I)-TEMP2(I)
      CALL DCOMP(NDOFS,TM,MAX),RETURNS(104)
      GO TO 105
  104 PRINT 106
  106 FORMAT(* ERROR IN DCOMP, M*)
  105 CONTINUE
      CALL SOLVE(NDOFS,TM,TA,D,MAX,MAXI,MAXI)
      DO 56 I=1,NDOFS
   56 DA(I)=D(I,MAXI)
C
C ----- COMPUTE SS
C
      DO 2 I=1,NDOFS
      DO 2 J=1,NDOFS
    2 SS(I,J)=S(I,J)+4*M(I,J)/(TINC*TINC)+2*C(I,J)/TINC
C
C ----- DECOMPOSE SS
C
      CALL DCOMP(NDOFS,SS,MAX),RETURNS(101)
      GO TO 103
  101 PRINT 102
  102 FORMAT(* ERROR IN DCOMP,SS*)
  103 CONTINUE
C
C ----- STEP PROCEDURE BEGINS
C
 301  CONTINUE
      NPTT=NPTS+1
      IF(NXTOT.EQ.MN) GO TO 4
      L=1
    4 DO 9 I=1,NDOFS
      P(I)=2*D(I,L)/TINC+DV(I)
    9 Q(I)=DA(I)+4*(DV(I)+D(I,L)/TINC)/TINC
C
```

```
C ----- CONSTRUCTION OF A
C
      IF(NXTOT.EQ.NN.AND.L.EQ.1) GO TO 5
      CALL CONSTA(NDOFS,MAX,NRA,LF,FXI,FYI,FZI,L,A,MAXI)
C
C ----- CALCULATION OF AS
C
    5 CALL MULTI(C,P,NDOFS,MAX,TEMP1)
      CALL MULTI(M,Q,NDOFS,MAX,TEMP2)
      DO 8 I=1,NDOFS
    8 A(I)=A(I)+TEMP1(I)+TEMP2(I)
C
C ----- SOLVE FOR  D
C
      L=L+1
      CALL SOLVE(NDOFS,SS,A,D,MAX,MAXI,L)
C
C ----- COMPUTE  DV AND DA
C
      DO 10 I=1,NDOFS
      DA(I)=4*D(I,L)/(TINC*TINC)-Q(I)
   10 DV(I)=2*D(I,L)/TINC-P(I)
      IF(L.LE.NPTS) GO TO 4
C
C ----- STEP PROCEDURE ENDS
C
C
C ------ WRITE  D  ON TAPE
C
      NC=2
      IF(NXTOT.EQ.NN) NC=1
      WRITE(23) (D(1,J),J=NC,NPTT)
      WRITE(24) (D(28,J),J=NC,NPTT)
      WRITE(25) (D(52,J),J=NC,NPTT)
      PRINT 601,(D(1,J),J=NC,NPTT)
  601 FORMAT(/9(F12.5))
      DO 303 I=1,NDOFS
  303 D(I,1)=D(I,NPTT)
      IF(NXTOT.LT.NXDATA) GO TO 300
C
      STOP
      END
```

```
*DECK MATPRT
      SUBROUTINE MATPRT (M,N,A,KA,LA)
C        THIS SUBROUTINE PRINTS AN MXN MATRIX, 9 COLUMNS AT A TIME
C        KAXLA IS THE DIMENSION OF THE MATRIX IN MAIN
      DIMENSION A(KA,LA)
      NP=(N-1)/9+1
      DO 1 I=1,NP
      LT=9*(I-1)+1
      LR=9*I
      IF(LR.GT.N) LR=N
      PRINT 2,(K,K=LT,LR)
    2 FORMAT(//* COLUMN *,I4,3X,9(I10,3X))
      PRINT 3
    3 FORMAT(* -------*/*  ROW*)
      DO 1 J=1,M
    1 PRINT 4,J,(A(J,K),K=LT,LR)
    4 FORMAT(* *,I3,1X,1P9E13.5)
      RETURN
      END
```

```
*DECK SOLVE
      SUBROUTINE SOLVE (N,U,B,X,MAX,MAXI,L)
C
C-----THIS SUBPROGRAM SOLVES THE SYSTEM OF SIMULTANEOUS EQUATIONS.
C     A * X = B, IN A TWO-STEP PROCESS AS DISCUSSED ON PAGE 57 OF
C     )COMPUTER PROGRAMS FOR STRUCTURAL ANALYSIS) BY WM. WEAVER, JR.
C     (1967).  SUBPROGRAM )DCOMP) PROVIDES THE DECOMPOSED VERSION OF
C     MATRIX A (STORED IN UPPER TRIANGULAR MATRIX U) NEEDED IN THE
C     SOLUTION PROCESS.  X IS THE VECTOR (OR MATRIX) OF UNKNOWNS AND
C    /B IS THE VECTOR (OR MATRIX) OF KNOWN QUANTITIES.
C
C
C
      DIMENSION U(MAX,MAX),B(MAX),X(MAX,MAXI)
C
      DO 101 I=1,N
      SUM=B(I)
      K1=I-1
      IF(I.EQ.1) GO TO 101
      DO 102 K=1,K1
 102  SUM=SUM-U(K,I)*X(K,L)
 101  X(I,L)=SUM*U(I,I)
C
      DO 103 I1=1,N
      I=N-I1+1
      SUM=X(I,L)
      K2=I+1
      IF(I.EQ.N) GO TO 103
      DO 104 K=K2,N
 104  SUM=SUM-U(I,K)*X(K,L)
 103  X(I,L)=SUM*U(I,I)
C
      RETURN
      END
```

```
*DECK CONSTA
      SUBROUTINE CONSTA(NDOFS,MAX,NRA,LF,FX,FY,FZ,L,A,MAXI)
C
C ----- THIS SUBROUTINE CONSTRUCTS THE A MATRIX
C
      DIMENSION NRA(MAX),A(MAX),FX(MAXI),FY(MAXI),FZ(MAXI)
      REAL LF(MAX)
C
      DO 5 I=1,NDOFS
      IF(NRA(I).NE.1) GO TO 6
      A(I)=LF(I)*FX(L)
      GO TO 5
    6 IF(NRA(I).NE.2) GO TO 7
      A(I)=LF(I)*FY(L)
      GO TO 5
    7 A(I)=LF(I)*FZ(L)
    5 CONTINUE
      RETURN
      END
```

```
*DECK DCOMP
      SUBROUTINE DCOMP (N,A,MAX1),RETURNS(JBJ)
C
C-----THIS SUBPROGRAM DECOMPOSES THE SYMMETRIC, POSITIVE-DEFINITE MATRIX
C     A USING THE CHOLESKY SQUARE ROOT METHOD AS DISCUSSED ON PAGE 56 OF
C     )COMPUTER PROGRAMS FOR STRUCTURAL ANALYSIS.) (1967) BY WM. WEAVER,
C     JR..  MATRIX A IS NXN, BUT MAY BE DIMENSIONED FOR MAX1 X MAX2 IN T
C                                                                     HE
C     CALLING PROGRAM.
C
C
      DIMENSION A(MAX1,MAX1)
C
      DO 101 I=1,N
      DO 101 J=I,N
      SUM=A(I,J)
      K1=I-1
      IF(I.EQ.1) GO TO 1
      DO 102 K=1,K1
 102  SUM=SUM-A(K,I)*A(K,J)
   1  IF(J.NE.I) GO TO 2
      IF(SUM.LE.0.) RETURN JBJ
      TEMP=1.0/SQRT(SUM)
      A(I,J)=TEMP
      GO TO 101
   2  A(I,J)=SUM*TEMP
 101  CONTINUE
C
      RETURN
      END
```

```
*DECK MULTI
      SUBROUTINE MULTI (A,B,KA,L,C)
      DIMENSION A(L,L),B(L),C(L)
      DO 1 I=1,KA
      S=0.
      DO 3 M=1,KA
    3 S=A(I,M)*B(M)+S
    1 C(I)=S
      RETURN
      END
```

```
*DECK INTERP
      SUBROUTINE INTERP(F,T,TINC,TMAX,NDATA,NPTS,FI,TI,TTOT)
C
C---DEFINITION OF VARIABLES
C         F - VECTOR OF DIGITIZED RECORD VALUES
C          FIRST DATA ITEM IN F IE F(1) MUST BE AT TIME T(1)=0.
C         T - VECTOR OF TIMES CORRESPONDING TO VALUES IN ARRAY F
C         TINC - TIME INTERVAL AT WHICH F IS TO BE DIGITIZED (USING
C               LINEAR INTERPOLATION)
C         TMAX - DURATION OF RECORD (I.E. LAST VALUE IN ARRAY T)
C         NDATA - TOTAL NUMBER OF INTERPOLATED POINTS IN ARRAY FI
C         FI - VECTOR CONTAINING INTERPOLATED VALUES OF F
C         TI - VECTOR CONTAINING TIMES CORRESPONDING TO ARRAY FI VALUES
C
      DIMENSION F(1),T(1),FI(1),TI(1)
      NPTS=0
C
C ----- CALCULATES A VALUE OF F AT MIDPOINT OF INTERVAL(SEE
C ----- VIBRATION PROBLEMS P.131 FIG.1.56)
C
      TT=-TINC/2.
      TTOT=-TINC/2.
      N=1
      SLOPE=(F(N+1)-F(N))/(T(N+1)-T(N))
    9 NPTS=NPTS+1
      TT=TT+TINC
      TTOT=TTOT+TINC
      IF(TTOT-T(N+1)-T(1))10,10,20
   20 TT=T(N+1)-T(N)
   10 IF(TTOT-T(N+1)-T(1))30,30,40
   30 FI(NPTS)=F(N)+SLOPE*TT
      TI(NPTS)=TTOT
   40 IF(TTOT-T(N+1)-T(1))50,50,60
   60 N=N+1
      TT=TTOT-T(N)-T(1)
      IF(TTOT-T(N+1)-T(1))70,70,80
   80 TT=T(N+1)-T(N)
   70 IF(N.LT.NDATA)GO TO 90
      GO TO 10
   90 SLOPE =(F(N+1)-F(N))/(T(N+1)-T(N))
      GO TO 10
   50 IF((TTOT+TINC).LE.TMAX) GO TO 9
      DO 1 I=1,NPTS
    1 TI(I)=TI(I)+TINC/2.
      TTOT=TTOT+TINC/2.
      RETURN
      END
```